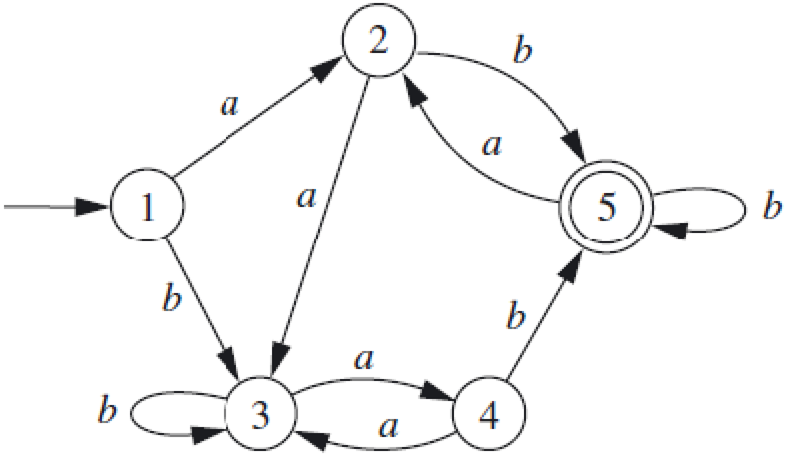
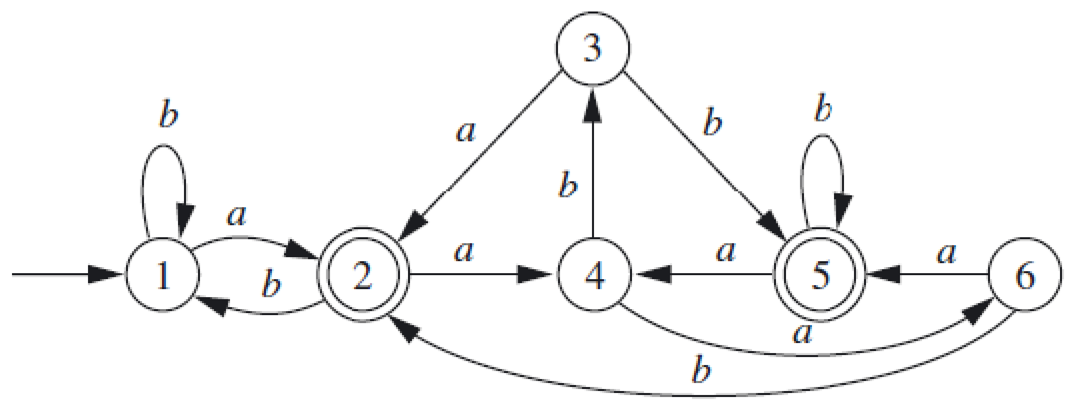
Az informatika számítástudományi alapjai gyakorlat

3. feladatsor

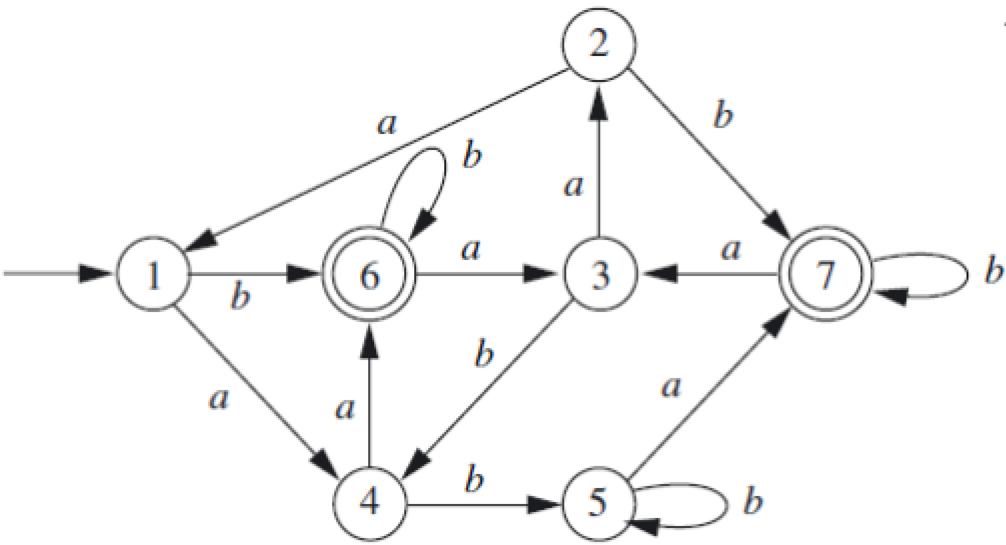
2.55. For each of the FAs pictured in Fig. 2.45, use the minimization algorithm described in Section 2.6 to find a minimum-state FA recognizing the same language. (It's possible that the given FA may already be minimal.)



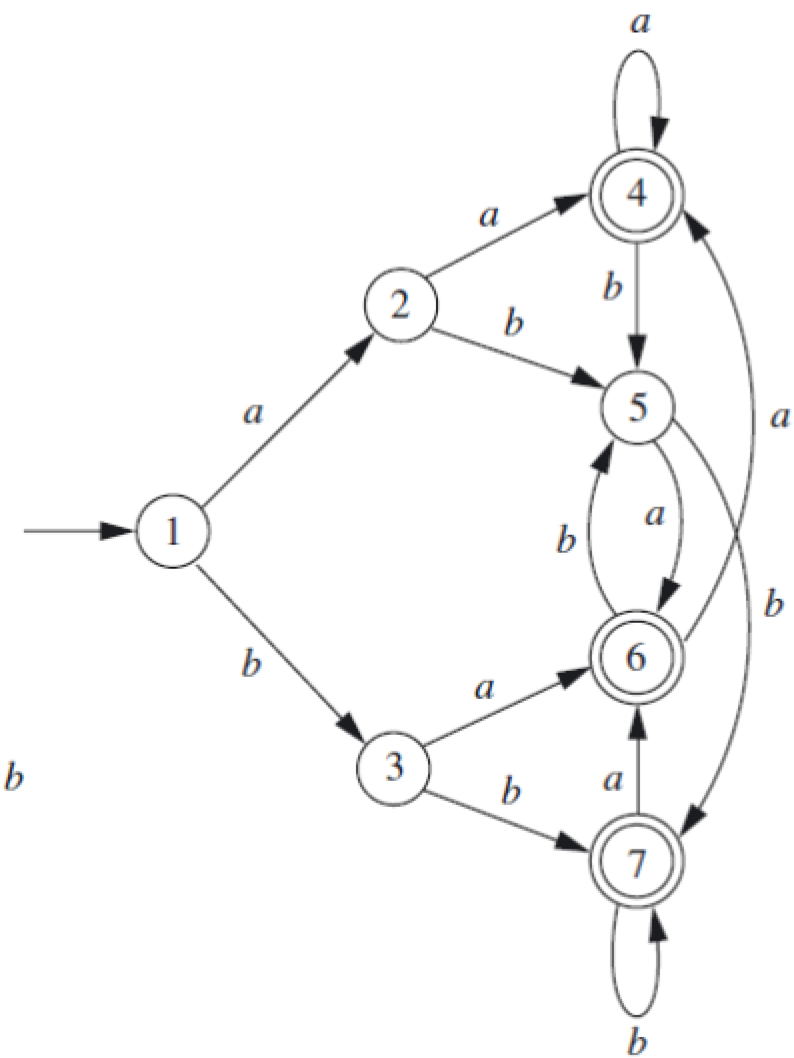
(*a*)



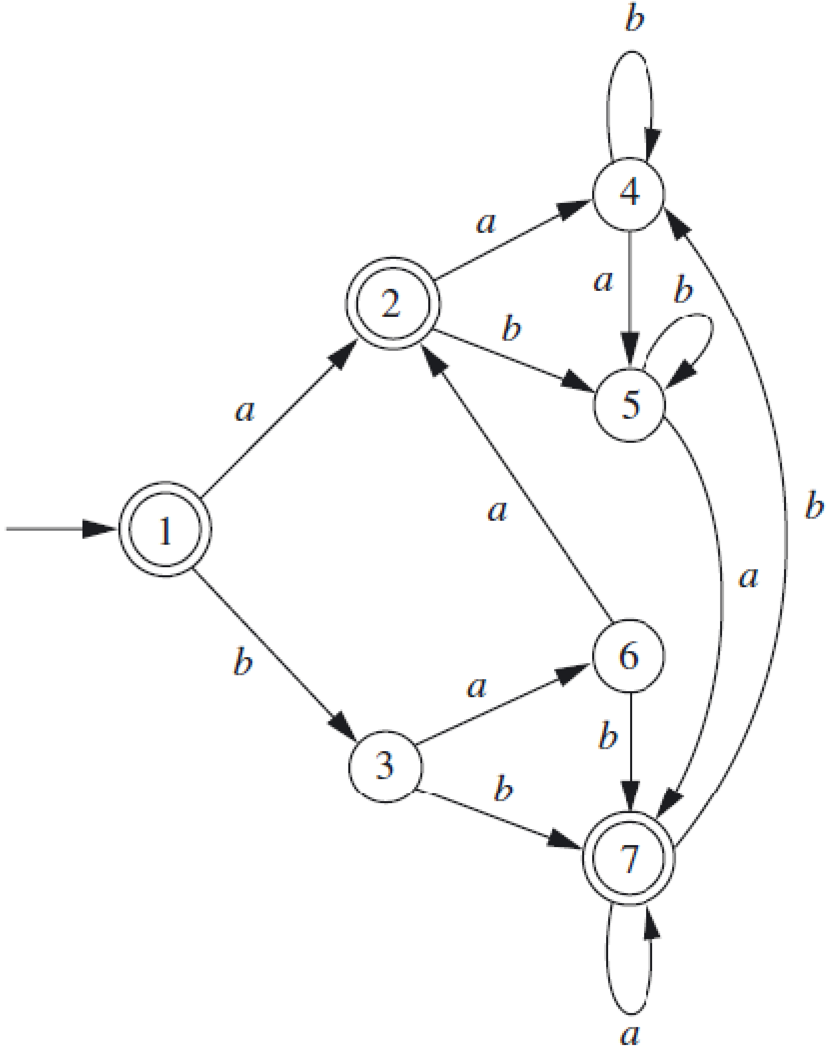
(*b*)



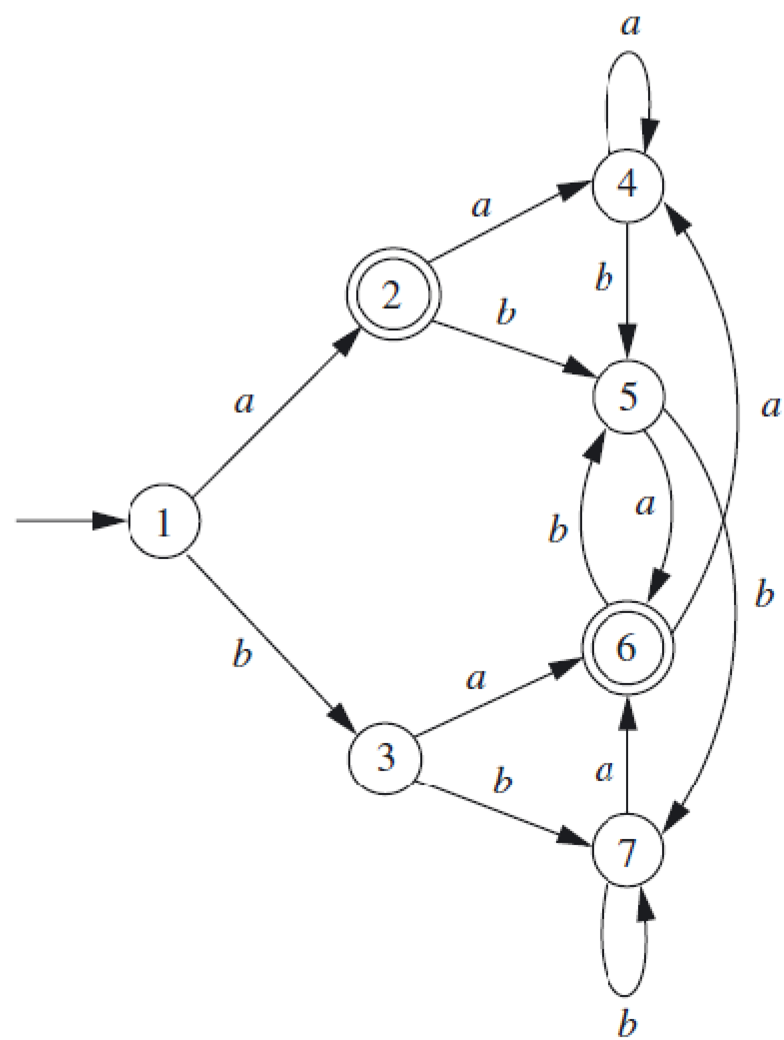
(c)



(d)



(e)



(f)

The Equivalence Classes of I_L, Where $L = AnBn$ — EXAMPLE 2.37

As we observed in Example 2.30, accepting $AnBn = \{a^n b^n \mid n \geq 0\}$ requires that we remember how many a's we have read, so that we can compare that number to the number of b's. A precise way to say this is that two different strings of a's are L-distinguishable: if $i \neq j$, then $a^i b^i \in L$ and $a^j b^i \notin L$. Therefore, the equivalence classes $[a^j]$ are all distinct. If we were interested only in showing that the set of equivalence classes is infinite, we could stop here.

Exactly what are the elements of $[a^j]$? Not only is the string a^j L-distinguishable from a^i, but it is L-distinguishable from *every* other string x: A string that distinguishes the two is ab^{j+1}, because $a^j ab^{j+1} \in L$ and $xab^{j+1} \notin L$. Therefore, there are no other strings in the set $[a^j]$, and

$$[a^j] = \{a^j\}$$

Each of the strings a^i is a prefix of an element of L. Other prefixes of elements of L include elements of L themselves and strings of the form $a^i b^j$ where $i > j$. All other strings in $\{a, b\}^*$ are nonprefixes of elements of L.

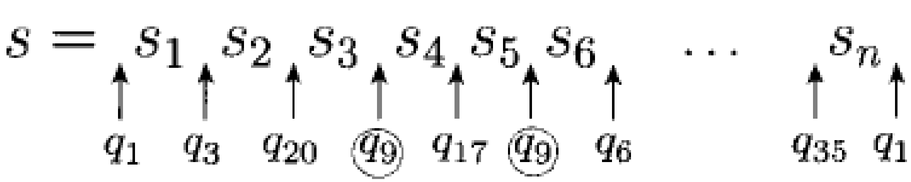
Two nonnull elements of L are L-indistinguishable, because if a string other than Λ is appended to the end of either one, the result is not in L; and every nonnull string in L can be distinguished from every string not in L by the string Λ. Therefore, the set $L - \{\Lambda\}$ is an equivalence class of I_L.

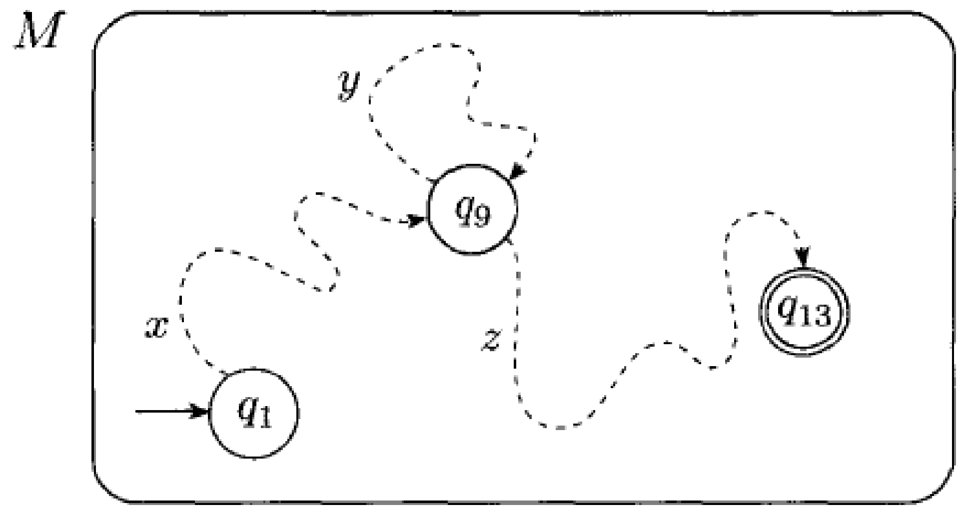
The set of nonprefixes of elements of L is another equivalence class: No two nonprefixes can be distinguished relative to L, and if $xy \in L$, then the string y distinguishes x from every nonprefix.

We are left with the strings $a^i b^j$ with $i > j > 0$. Let's consider an example, say $x = a^7 b^3$. The only string z with $xz \in L$ is b^4. However, there are many other strings y that share this property with x; every string $a^{i+4} b^i$ with $i > 0$ does. The equivalence class $[a^7 b^3]$ is the set $\{a^{i+4} b^i \mid i > 0\} = \{a^5 b, a^6 b^2, a^7 b^3, \ldots\}$. Similarly, for every $k > 0$, the set $\{a^{i+k} b^i \mid i > 0\}$ is an equivalence class.

Pumpálási lemma

Ha egy véges automata által elfogadott szó eléghosszú, akkor az automata kénytelen legalább egy állapotot többször is felvenni.





Azaz:

Ha $L \subseteq \Sigma^*$ nyelvet elfogadja $M = (Q, \Sigma, q_0, A, \delta)$ véges automata és $n = |Q|$, akkor minden olyan $x \in L$ L-beli szó, amely $|x| \geq n$, felírható

$$x = uvw$$

alakban, ahol:

1. $|uv| \leq n$
2. $|v| > 0$ (azaz $v \neq \lambda$)
3. Minden $i \geq 0$-ra, $uv^iw \in L$

2.22. For each of the languages in Exercise 2.21, use the pumping lemma to show that it cannot be accepted by an FA.

a. $L = \{a^n b a^{2n} \mid n \geq 0\}$
b. $L = \{a^i b^j a^k \mid k > i + j\}$
c. $L = \{a^i b^j \mid j = i \text{ or } j = 2i\}$
d. $L = \{a^i b^j \mid j \text{ is a multiple of } i\}$
e. $L = \{x \in \{a, b\}^* \mid n_a(x) < 2n_b(x)\}$
f. $L = \{x \in \{a, b\}^* \mid \text{no prefix of } x \text{ has more b's than a's}\}$
g. $L = \{a^{n^3} \mid n \geq 1\}$
h. $L = \{ww \mid w \in \{a, b\}^*\}$

2.29. For each statement below, decide whether it is true or false. If it is true, prove it. If it is not true, give a counterexample. All parts refer to languages over the alphabet $\{a, b\}$.

a. If $L_1 \subseteq L_2$, and L_1 cannot be accepted by an FA, then L_2 cannot.

b. If $L_1 \subseteq L_2$, and L_2 cannot be accepted by an FA, then L_1 cannot.

c. If neither L_1 nor L_2 can be accepted by an FA, then $L_1 \cup L_2$ cannot.

e. If L cannot be accepted by an FA, then L' cannot.