Name: Neptun ID: Date:

Probability Theory and Mathematical Statistics
Applied Statistics
Retake1+2

1. In an urn we have three red balls. Find the minimal number of white balls to be added to have the probability of choosing a white ball be greater than 0.9. [5 points]
2. We know that at least one of the two kids in a family is a girl. Find the probability of having also a boy in the family. [5 points]
3. 75% of the products produced in a factory are first class products, the rest are of second class. Later all products are double checked. The probability that a first class product is evaluated as a second class one is 0.02, while the probability that a second class product is found to be a first class one is 0.05. Find the probability that a product evaluated as a first class product is really a first class one. [5 points]
4. The joint cdf of (ξ, η) equals:

$$F(x) = \begin{cases} 1 + e^{-x-y} - e^{-x} - e^{-y} & if\ x > 0,\ \ y > 0, \\ 0, & otherwise. \end{cases}$$

   a. Find the marginal distribution functions of ξ and η.

   Are ξ and η independent? [10 points]
5. The joint pdf of (ξ, η) equals:

$$\mathrm{f(x,y)} := \begin{cases} \frac{4}{5}(\mathrm{x} + \mathrm{xy} + \mathrm{y}), & \mathrm{ha}\ 0 < \mathrm{x} < 1, 0 < \mathrm{y} < 1, \\ 0, & \mathrm{otherwise}. \end{cases}$$

   b. Find the marginal density functions of ξ and η.

   c. Are ξ and η independent? [10 points]
6. Check whether the following function is a probability density function or not. If it is a pdf find the mean and standard deviation of the random variable specified by the following probability density function:

$$f(x) = \begin{cases} \frac{1}{x}, if\ 1 < x < e, \\ 0, otherwise; \end{cases}$$ [5 points]

7. Write down the cdf, pdf, expected value and variance of the Uniform distribution! [10 points]
8. Write down the cdf, pdf, expected value and variance of the Exponential distribution! [10 points]
9. Write down the Bayes' theorem! [10 points]
10. Write down the definition of independence! [10 points]