

Seminar 3
Real numbers

+ Mathematical induction!

1. Prove the following statements!

- $1 + 2 + \dots + n = \frac{n(n+1)}{2} \quad \forall n \in \mathbb{N}$

- $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1} \quad \forall n \in \mathbb{N}$

- $1 \cdot 1! + 2 \cdot 2! + \dots + n \cdot n! = (n+1)! - 1 \quad \forall n \in \mathbb{N}$

- $6|(n^3 - n) \quad \forall n \in \mathbb{N}$

- $5|(2^{4n+1} + 3) \quad \forall n \in \mathbb{N}$

+ Divisibility!

2. Prove the following divisibilities!

- $9|(10^{19} + 53)$

- $6|(10^7 - 88)$

- $36|(10^{17} - 64)$

- $12|(10^{16} + 44)$

+ Absolute value

3. What is the absolute value of

67	6
-65	-15
34	-92

4. Solve the equations

$$|x - 1| = 2$$

$$|3 - x| = -1$$

$$|8x - 40| = 0$$

$$|x - 1| < 2$$

$$|x - 3| = 2x + 5$$

5. Prove that $||x| - |y|| \leq |x - y|!$