App. math. test

March 25, 2019

Each question has exactly **one** correct answer. Please circle exactly **one** answer indicating your choice. Points for correct answers can be found next to the questions, there will be -1 point penalty for incorrect or ambigously answered questions. Evaulating is done by the following system:

result (%)	<39	40-54	55-69	70-84	>85
mark	1	2	3	4	

1 Norms

Consider

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -3 & 5 \\ -1 & 4 & 0 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

1. The quantities $||x||_1, ||Ax||_\infty, ||x||_2, ||A||_1, ||A||_\infty$ are: **(5)**

   A. $5, 2, \sqrt{2}, 8, 10$ B. $2, 5, \sqrt{5}, 8, 10$

   C. $2, 5, \sqrt{2}, 8, 10$ D. $2, 5, \sqrt{2}, 8, 11$

2. For all $v \in \mathbb{R}^3$ we have: **(2)**

   A. $||Av||_\infty \leq 10||v||_\infty$ B. $||Av||_\infty < 10||v||_\infty$

   C. $||Av||_\infty \geq 10||v||_\infty$ D. $||Av||_\infty > 10||v||_\infty$

2 Floats

Consider the floating point number system

$$\mathcal{F} = [a = 2, k_- = -5, k_+ = 6, t = 4].$$

1. The quantities $\varepsilon_0, M_\infty, 1_+$ and the number of positive normalized elements in $\mathcal{F}$ are: **(4)**

   A. $\frac{1}{32}, 64, \frac{9}{8}, 96$ B. $\frac{1}{64}, 60, \frac{17}{16}, 96$

   C. $\frac{1}{64}, 60, \frac{9}{8}, 96$ D. $\frac{1}{32}, 60, \frac{9}{8}, 88$

2. The representation of 0.345 (using regular rounding) and its right neighbour in $\mathcal{F}$: (**3**)

   A. 2^{-1} 0.1010 and 2^{-1} 0.1011   B. 2^{-1} 0.1011 and 2^{-1} 0.1100

   C. 2^{0} 0.1011 and 2^{-1} 0.1100   D. 2^{-1} 0.1011 and 2^{0} 0.1000

3 LSA

Consider the point set P

$$(2,0),\ (2,2),\ (-1,2),\ (-1,1),\ (-1,-1)$$

1. The LSA line for P and its value at 1: (**4**)

   A. $\frac{1}{9}t - \frac{5}{9}$ and $-\frac{4}{9}$ B. $\frac{7}{9}t - \frac{1}{9}$ and $\frac{6}{9}$

   C. $\frac{1}{9}t + \frac{7}{9}$ and $\frac{8}{9}$ D. $\frac{13}{9}t + \frac{7}{9}$ and $\frac{20}{9}$

2. For the model $F(t) = x_1 + x_2 \cos(\pi t)$ the matrix A in the Gauss normal equation is: (**3**)

   A. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$

4 Factorization

Consider the matrix A and vector b:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 1 & -1 \\ 2 & -3 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Find the LU factorization and the determinant of A! Solve $Ax = b$!

1. The solution of $Ly = b$ and $Ux = y$ and $det(A)$ are respectively: (**7**)

   A. $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -7 \\ -7 \\ -2 \end{bmatrix}$, 2.   B. $\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} -7 \\ -7 \\ -2 \end{bmatrix}$, -2.

   C. $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -7 \\ -2 \end{bmatrix}$, 2.   D. $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 7 \\ -7 \\ -2 \end{bmatrix}$, -2.

5 Interpolation

Consider the point set

$$Q = (-2,-13),\ (-1,-4),\ (1,2).$$

Find the p Lagrange interpolating polynomial for Q!

1. $p(t) =$ **(4)**

   A. $2 + 9(t+2) - 2(t+2)(t+1)$ B. $13 + 9(t+2) - 2(t+2)(t+1)$

   C. $2 + 3(t-1) - 2(t-1)(t+1)$ D. $2 + 3(t-1) + 2(t-1)(t+1)$

   Let us extend Q by a new point $(0, 3)$:

   $$R = (-2, -13),\ (-1, -4),\ (1, 2),\ (0, 3).$$

   Find the $q(t)$ Lagrange interpolating polynomial for R!

2. $q(t) =$ **(3)**

   A. $3+8(t+2)-2(t+2)(t+1)+(t+1)(t+2)(t-1)$ B. $3+8(t+2)-2(t+2)(t+1)-(t+1)(t+2)(t-1)$

   C. $2+3(t-1)+2(t-1)(t+1)+(t-1)(t+1)(t+2)$ D. $2+3(t-1)-2(t-1)(t+1)-(t-1)(t+1)(t+2)$

6 Octave

1. Without using of computer, determine the value of v, after executing the commands: **(2)**

```
v=1:5
for i=1:4
  v(i)=v(i)-v(i+1)
end
```

   A. $\begin{bmatrix} -1 & -1 & -1 & 5 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$

   C. $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \end{bmatrix}$

2. Without using of computer, determine the value of w, after executing the commands: **(5)**

```
w=10:-1:1
for i=2:9
  w(i)=2*w(i+1)+w(i)-w(i-1)
end
```

   A. $\begin{bmatrix} 10 & 15 & 7 & 12 & 4 & 9 & 1 & 6 & -2 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 10 & 15 & 7 & 12 & 4 & 9 & 1 & 6 & 2 & 1 \end{bmatrix}$

   C. $\begin{bmatrix} 10 & 15 & 7 & 12 & 4 & 9 & 1 & 5 & 2 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 10 & 15 & 7 & 12 & 4 & 9 & 1 & 6 & 2 & 0 \end{bmatrix}$

7 Theo

1. For n points $(t_1, f_1), \ldots, (t_n, f_n)$ in the plane,
   Lagrange theorem on interpolating polynomials states: **(4)**

   A. There is exactly one polynomial p of degree no more than $n-1$,
   for which $p(t_i) = f_i, \quad i = 1, \ldots, n$.

B. If $t_i \neq t_j$ for $i \neq j$, then there is exactly one polynomial p of degree at most $n-1$, for which $p(t_i) = f_i, \quad i = 1, \ldots, n,$.

C. If $t_i \neq t_j$ for $i \neq j$, then there is exactly one polynomial p of degree at most n, for which $p(t_i) = f_i, \quad i = 1, \ldots, n,$.

D. There is exactly one polynomial p of degree at least $n-1$ for which $p(t_i) = f_i, \quad i = 1, \ldots, n.$

2. For $n > 0$ points $(t_1, f_1), \ldots, (t_n, f_n)$ in the plane, the linear LSA problem: (**3**)

A. has always at most one solution.

B. has always at least one solution.

C. has always infinitely many solutions.

D. has no solution at all, for some point set.