Formal Languages

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- *$L_\emptyset$ is the empty language (it has no any word).*

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  - $L^{n+1} = L^n \circ L$
- *Transitive closure (Kleene-star operation):* $L^\star = \cup_{i=0}^{\infty} L^i$, *i.e.*

$$L^\star = \{x_1 x_2 \dots x_k \mid k \geq 0 \text{ és } x_i \in L\}$$

Remark

- $L_\varepsilon^\star = \{\varepsilon\}$, *ahol* $L_\varepsilon = \{\varepsilon\}$.

Examples

Let $\Sigma = \{a, b, c\}$, $L_1 = \{a, bb, ccc\}$ and $L_2 = \{ac, bc\}$. Then

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- $L_1^\star = \{\varepsilon, a, bb, ccc, aa, abb, accc, bba, bbbb, bbccc, \dots\}$;
- Σ is a formal language over itself, and its transitive closure is $\Sigma^\star$ (i.e. the set of words over the abc Σ.

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Remark

The formal possibilities are useful in giving formal languages with different properties.

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- $0^* + (0^* \cdot 1 \cdot 0^* \cdot 1 \cdot 0^* \cdot 1 \cdot 0^* \cdot 1 \cdot 0^* \cdot 1 \cdot 0^*)^*$

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6. *If $R \in \mathcal{R}$, then $R^\star \in \mathcal{R}$.*

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Let $\mathcal{R}$ be the set of regular expressions over abc Σ and $R \in \mathcal{R}$. The language described by the regular expression R is given by the following inductive definition:

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Theorem

If languages L_1 és L_2 over abc Σ are regular, then languages $L_1 \cup L_2$, $L_1 \cap L_2$, $L_1 \backslash L_2$, $\overline{L_1} (= \Sigma^\star \backslash L_1)$, $L_1 \circ L_2$ és $L_1^\star$ are regular.

Finite automata

- The simplest automaton: it is only able to read and it has no memory.

Definition

A finite automaton is an ordered 5-tuple: $A = \langle Q, \Sigma, \delta, q_0, F \rangle$, *where*

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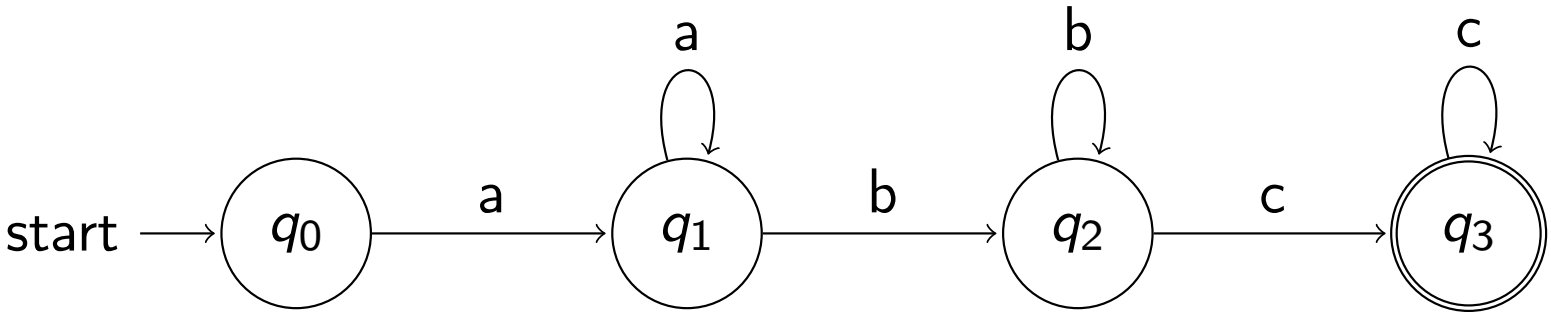

$a^i b^j c^k$, ahol $i, j, k > 0$

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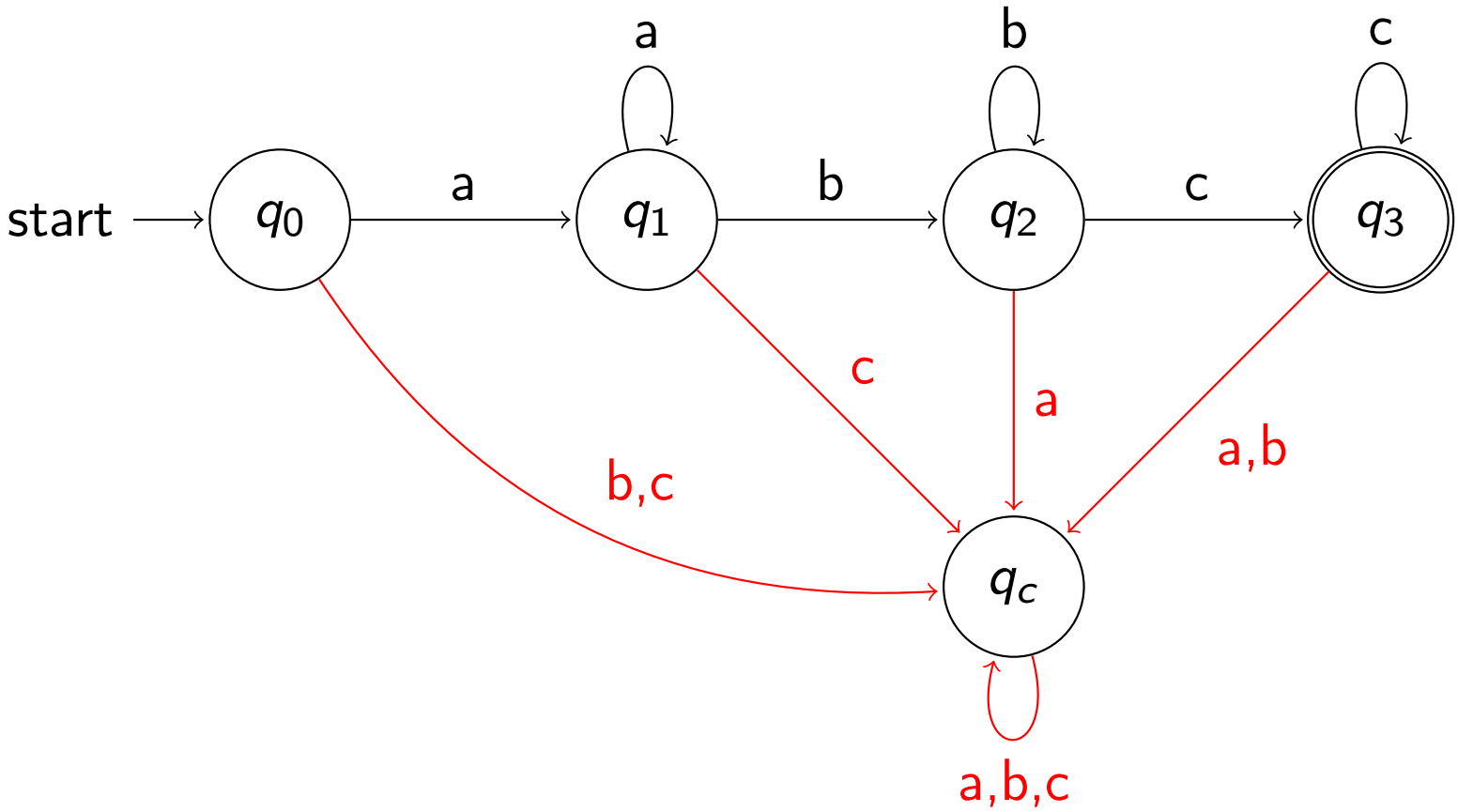

- Internal states are represented by circles, the transition function is denoted by arrows and the characters of internal abc. The initial state is the state with an arrow without a character. The final states is the doubled circle.

Accepting automata

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$A = \langle Q, \Sigma, \delta, q_0, F \rangle$ is an automaton, and $w = a_1 a_2 \dots a_n \in \Sigma^\star$ is a word. The automaton accepts the word w if

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Theorem

Finite automata accept regular languages, i.e. if L is a regular language, then there is an automaton A such that $L = L_A$