Definitions

1. Define the first-order term!
2. Define the first-order atomic formula!

Practical exercises

Let $L^{(1)}$ be a first-order language:

$$L^{(1)} = \langle LC, \{x_1, x_2, \ldots\}, \{p, q, f, g, c\}, Term, Form\rangle$$

- $p \in P(1)$ and $q \in P(2)$ are predicate parameters,
- $f \in F(1)$ and $g \in F(2)$ are function parameters,
- $c \in F(0)$ is a name parameter.

Let $\langle U, \varrho\rangle$ be an interpretation of $L^{(1)}$:

- $U = \{1, 2, 3, 4\}$
- $\varrho(p) = p'$ $\quad p'(x) = \begin{cases} 1 & \text{if } x = 1 \text{ or } x = 4 \\ 0 & \textit{otherwise} \end{cases}$
- $\varrho(q) = q'$ $\quad q'(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \textit{otherwise} \end{cases}$
- $\varrho(f) = f'$ $\quad f'(x) = 5 - x,$
- $\varrho(g) = g'$ $\quad g'(x, y) = \text{abs}(x - y) + 1$
- $\varrho(c) = 2$

Evaluate the followings:

1. if $v(x_1) = 1, v(x_2) = 3$

   (a) $|c|_v^{\langle U,\varrho\rangle}$

   (b) $|x_2|_v^{\langle U,\varrho\rangle}$

   (c) $|f(c)|_v^{\langle U,\varrho\rangle}$

   (d) $|f(g(c, c))|_v^{\langle U,\varrho\rangle}$

   (e) $|g(f(x_1), f(x_2))|_v^{\langle U,\varrho\rangle}$

   (f) $|f(g(x_1, g(x_2, c)))|_v^{\langle U,\varrho\rangle}$

2. if $v(x_1) = 2, v(x_2) = 4$

   (a) $|x_2|_v^{\langle U,\varrho\rangle}$
   (b) $|g(f(x_1), f(x_2))|_v^{\langle U,\varrho\rangle}$
   (c) $|f(g(x_1, g(x_2, c)))|_v^{\langle U,\varrho\rangle}$

3. if $v(x_1) = 1, v(x_2) = 3$

   (a) $|p(x_2)|_v^{\langle U,\varrho\rangle}$
   (b) $|\forall x_1 p(x_1)|_v^{\langle U,\varrho\rangle}$
   (c) $|q(x_1, x_2)|_v^{\langle U,\varrho\rangle}$
   (d) $|q(x_1, x_2) \supset \neg q(f(x_1), f(x_2))|_v^{\langle U,\varrho\rangle}$
   (e) $|\forall x_1(q(f(x_1), x_2) \supset \neg q(x_1, f(x_2)))|_v^{\langle U,\varrho\rangle}$
   (f) $|\exists x_2(q(f(c), x_2) \supset \neg q(c, f(x_2)))|_v^{\langle U,\varrho\rangle}$
   (g) $|\exists x_1 p(x_1) \supset \forall x_1 \neg q(x_1, f(x_1))|_v^{\langle U,\varrho\rangle}$
   (h) $|\forall x_1 \exists x_2 \neg q(x_1, f(x_2))|_v^{\langle U,\varrho\rangle}$
   (i) $|\exists x_1 \forall x_2 \neg q(x_1, f(x_2))|_v^{\langle U,\varrho\rangle}$
   (j) $|\forall x_1 \forall x_2(q(x_1, f(x_2)) \vee q(f(x_2), x_1))|_v^{\langle U,\varrho\rangle}$

3. Prenex form of:

   1. $\forall x_1 q(f(x_1), x_2) \supset \neg q(x_1, f(x_2))$
   2. $\exists x_2 q(f(c), x_2) \supset \neg q(c, f(x_2))$
   3. $\exists x_1 p(x_1) \supset \forall x_1 \neg q(x_1, f(x_1))$
   4. $\neg \forall x_1 \exists x_2 q(x_1, f(x_2))$
   5. $\neg \exists x_1 \forall x_2 q(x_1, f(x_2))$
   6. $\forall x_1 \forall x_2(q(x_1, f(x_2)) \vee q(f(x_2), x_1))$

4. Construction tree of:

   1. $\forall x_1 q(f(x_1), x_2) \supset \neg q(x_1, f(x_2))$
   2. $\exists x_2 q(f(c), x_2) \supset \neg q(c, f(x_2))$

3. $\exists x_1 p(x_1) \supset \forall x_1 \neg q(x_1, f(x_1))$
4. $\neg \forall x_1 \exists x_2 q(x_1, f(x_2))$
5. $\neg \exists x_1 \forall x_2 q(x_1, f(x_2))$
6. $\forall x_1 \forall x_2 (q(x_1, f(x_2)) \vee q(f(x_2), x_1))$