

## Definitions

1. Complete the following rule:

$$\frac{\Gamma \vdash (A \supset B); \quad ?}{\Gamma \vdash B}$$

2. Complete the following rule:

$$\frac{\Gamma, A \vdash B; \quad ?}{\Gamma \vdash \neg A}$$

3. Complete the following rule:

$$\frac{\Gamma \vdash A; \quad ?}{\Gamma \vdash (A \wedge B)}$$

4. Complete the following rule:

$$\frac{\Gamma, B \vdash A; \quad ?}{\Gamma \vdash (A \equiv B)}$$

5. Complete the following rule:

$$\frac{\Gamma \vdash (A \equiv B); \quad ?}{\Gamma \vdash A}$$

6. Complete the following rule:

$$\frac{?}{\Gamma, A \vdash A}$$

7. Complete the following rule:

$$\frac{\Gamma, A \vdash C; \quad ?}{\Gamma, (A \vee B) \vdash C}$$

8. Complete the following rule:

$$\frac{\Delta, A \vdash B; \quad ?}{\Gamma, \Delta \vdash B}$$

## Practical exercises

9. Prove the correctness of

$$\frac{\Gamma \vdash A}{\Gamma \vdash (A \vee B)}$$

10. Prove the correctness of

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash (A \supset B)}$$

11. Prove the correctness of

$$\frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}$$

12. Prove the validity of  $p \wedge q \supset \neg\neg p$  using the natural deduction.

13. Prove the validity of  $\neg\neg p \supset (q \supset p)$  using the natural deduction.

14. Prove the validity of  $\neg(p \wedge (p \supset q) \wedge (p \supset \neg q))$  using the natural deduction.

15. Prove the validity of  $\neg((q \supset \neg p) \wedge (p \wedge q))$  using the natural deduction.

16. Prove the validity of  $\neg\neg p \wedge q \supset p$  using the natural deduction.

17. Prove the validity of  $p \supset (q \supset \neg\neg p)$  using the natural deduction.

18. Prove the validity of  $\neg((\neg q \supset p) \wedge (\neg p \wedge \neg q))$  using the natural deduction.