Definitions

1. Define the following: the model of a Γ set.

Let $\Gamma \subseteq Form$ be a set of formulas. An interpretation ϱ is a model of the set of formulas Γ, if $|A|_\varrho = 1$ for all $A \in \Gamma$.

2. Define the following: the model of a formula.

A model of a formula A is the model of the singleton $\{A\}$.

3. Define the following: the satisfiable a set of formulas.

The set of formulas $\Gamma \subseteq Form$ is satisfiable if it has a model. (If there is an interpretation in which all members of the set Γ are ture.)

4. Define the following: the satisfiable formula

A formula $A \in Form$ is satisfiable, if the singleton $\{A\}$ is satisfiable.

5. Define the following: unsatisfiable set

The set $\Gamma \subseteq Form$ is unsatisfiable if it is not satisfiable.

6. Define the following: unsatisfiable formula

A formula $A \in Form$ is unsatisfiable if the singleton $\{A\}$ is unsatisfiable.

7. Define the following: logical consequence

A formula A is the logical consequence of the set of formulas Γ if the set $\Gamma \bigcup \{\neg A\}$ is unsatifiable. (Notation: $\Gamma \models A$)

8. Define the following: valid formula.

The formula A is valid if $\emptyset \models A$. (Notation: $\models A$)

9. Define the following: logical equivalence

The formulas A and B are logically equivalent if $A \models B$ and $B \models A$. (Notation: $A \Leftrightarrow B$)

Practical exercises

10. Prove the equivalence: $p \supset q \Leftrightarrow \neg p \vee q$.

$p\ q$	p	$\supset$	q	$\neg$	p	$\vee$	q
0 0		1		1		1	
0 1		1		1		1	
1 0		0		0		0	
1 1		1		0		1	

They are equal.

11. Prove the equivalence: $p \supset q \Leftrightarrow \neg q \supset p$.

$p\ q$	p	$\supset$	q	$\neg$	q	$\supset$	p
0 0		1		1		0	
0 1		1		0		1	
1 0		0		1		1	
1 1		1		0		1	

They are not equal.

12. Prove the equivalence: $\neg(p \wedge q) \Leftrightarrow \neg p \vee q$.

$p\ q$	$\neg$	$(p \wedge q)$	$\neg$	p	$\vee$	q
0 0	1	0	1		1	
0 1	1	0	1		1	
1 0	1	0	0		0	
1 1	0	1	0		1	

They are not equal.

13. Prove the equivalence: $\neg(p \vee q) \Leftrightarrow \neg p \wedge q$.

$p\ q$	$\neg$	$(p \vee q)$	$\neg$	p	$\wedge$	q
0 0	1	0	1		0	
0 1	0	1	1		1	
1 0	0	1	0		0	
1 1	0	1	0		0	

They are not equal.

14. Are the following formula valid: $p \supset q \supset \neg p \vee q$?

$p\ q$	p	$\supset$	q	$\supset$	$\neg$	p	$\vee$	q
0 0		1		1	1		1	
0 1		1		1	1		1	
1 0		1		1	0		0	
1 1		1		1	0		1	

It is valid.

15. Are the following formula valid: $p \supset q \supset \neg q \supset p$?

$p\ q$	p	$\supset$	q	$\supset$	$\neg$	p	$\supset$	q
0 0		1		1	1		0	
0 1		1		1	1		1	
1 0		1		1	0		1	
1 1		1		1	0		1	

It is valid.

16. Are the following formula valid: $\neg(p \wedge q) \supset \neg p \vee q$?

$p\ q$	$\neg$	$(p$	$\wedge$	$q)$	$\supset$	$\neg$	p	$\vee$	q
0 0	1		0		1	1		1	
0 1	1		0		1	1		1	
1 0	1		0		0	0		0	
1 1	0		1		1	0		1	

It is not valid.

17. Are the following formula valid: $\neg(p \vee q) \supset \neg p \wedge q$?

$p\ q$	$\neg$	$(p$	$\vee$	$q)$	$\supset$	$\neg$	p	$\wedge$	q
0 0	1		0		0	1		0	
0 1	0		1		1	1		1	
1 0	0		1		1	0		0	
1 1	0		1		1	0		0	

It is not valid.

18. Is q the logical consequence of $\neg q \supset p$ and $q \vee \neg p$?

1. $\neg q \supset p, q \vee \neg p \models q$ if and only if $\{\neg q \supset p, q \vee \neg p, \neg q\}$ unsatisfiable,
2. $\{\neg q \supset p, q \vee \neg p, \neg q\}$ satisfiable if
   $|\neg q \supset p|_\varrho = 1$ and $|q \vee \neg p|_\varrho = 1$ and $|\neg q|_\varrho = 1$,
3. if $|\neg q|_\varrho = 1$, then $|q|_\varrho = 0$,
4. if $|q \vee \neg p|_\varrho = 1$ but $|q|_\varrho = 0$, then $|\neg p|_\varrho = 1$,
5. if $|\neg p|_\varrho = 1$, then $|p|_\varrho = 0$,
6. if $|\neg q|_\varrho = 1$ and $|p|_\varrho = 0$, then $|\neg p \supset q|_\varrho = 0$,
7. but $|\neg p \supset q|_\varrho = 1$ and $|\neg p \supset q|_\varrho = 0$ is a contradiction!

The logical consequence holds.

19. Is q the logical consequence of $q \vee \neg p$ and $\neg q \supset \neg p$?

1. $q \vee \neg p, \neg q \supset \neg p \models q$ if and only if $\{q \vee \neg p, \neg q \supset \neg p, \neg q\}$ unsatisfiable,
2. $\{q \vee \neg p, \neg q \supset \neg p, \neg q\}$ satisfiable if $|q \vee \neg p|_\varrho = 1$ and $|\neg q \supset \neg p|_\varrho = 1$ and $|\neg q|_\varrho = 1$,
3. if $|\neg q|_\varrho = 1$, then $|q|_\varrho = 0$,
4. if $|q \vee \neg p|_\varrho = 1$ but $|q|_\varrho = 0$, then $|\neg p|_\varrho = 1$,
5. if $|\neg p|_\varrho = 1$, then $|p|_\varrho = 0$,
6. if $|q|_\varrho = 0$ and $|p|_\varrho = 0$, then $|q \vee \neg p|_\varrho = 1$ and $|\neg q \supset \neg p|_\varrho = 1$ and $|\neg q|_\varrho = 1$, so the set is satisfiable!

The logical consequence not holds.

20. What is the DNF and CNF of $\neg(q \supset p) \wedge r$

- $\neg(q \supset p) \wedge r \quad \Leftrightarrow$
- $\neg(\neg q \vee p) \wedge r \quad \Leftrightarrow$
- $\neg\neg q \wedge \neg p \wedge r \quad \Leftrightarrow$
- DNF+CNF: $q \wedge \neg p \wedge r$

21. What is the DNF and CNF of $(q \equiv p) \wedge r$

- $(q \equiv p) \wedge r \quad \Leftrightarrow$
- $(q \supset p) \wedge (p \supset q) \wedge r \quad \Leftrightarrow$
- CNF: $(\neg q \vee p) \wedge (\neg p \vee q) \wedge r \quad \Leftrightarrow$
- DNF: $(\neg q \wedge \neg p \wedge r) \vee (p \wedge q \wedge r)$