Definitions

1. Define the classical zero-order language!

The classical zero-order language is an ordered triple

$$L(0) = \langle LC; Con; Form \rangle$$

where

1. $LC = \{\neg; \supset; \wedge; \vee; \equiv; (;)\}$ (the set of logical constants).
2. $Con \neq \emptyset$; the countable set of non-logical constants (propositional parameters)
3. $LC \backslash Con = \emptyset$;
4. The set of formulae i.e. the set Form is given by the following inductive definition:
   - $Con \subset Form$
   - If $A \in Form$, then $\neg A \in Form$.
   - If $A; B \in Form$, then $(A \wedge B) \in Form$, $(A \vee B) \in Form$, $(A \supset B) \in Form$, $(A \equiv B) \in Form$.

2. Define the direct subformula!

- If A is an atomic formula, then it has no direct subformula;
- $\neg A$ has exactly one direct subformula: A;
- direct subformulae of formulae $(A \wedge B)$, $(A \vee B)$, $(A \supset B)$, $(A \equiv B)$ are formulae A and B, respectively.

3. Define the set of subformulae!

The set of subformulae of formula A – denoting: $SF(A)$ – is given by the following inductive definition:

1. $A \in SF(A)$
   (i.e. the formula A is a subformula of itself);

2. if $A' \in SF(A)$ and B is a direct subformula of A', then $B \in SF(A)$
(i.e., if A' is a subformula of A, then all direct subformulae of A' are subformulae of A).

4. Define the construction tree!

The contruction tree of a formula A is a finite ordered tree whose nodes are formulae,

- the root of the tree is the formula A,
- the node with formula $\neg B$ has one child: the node with the formula B,
- the node with formulae $(B \vee C)$, $(B \wedge C)$, $(B \supset C)$, $(B \equiv C)$ has two children: the nodes with B, and C the leaves of the tree are atomic formulae.

Practical part

5. Logical degree of $p \wedge q \supset \neg p \equiv \neg q$.

5

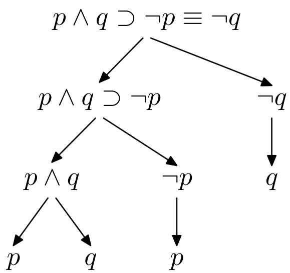
6. Direct subformulas of $p \wedge q \supset \neg p \equiv \neg q$.

$p \wedge q \supset \neg p$, and $\neg q$

7. Set of subformulas of $p \wedge q \supset \neg p \equiv \neg q$.

$$\{p \wedge q \supset \neg p \equiv \neg q, p \wedge q \supset \neg p, \neg q, p \wedge q, \neg p, q, p\}$$

8. Construction tree of $p \wedge q \supset \neg p \equiv \neg q$.



9. Logical degree of $p \wedge (q \supset \neg p) \equiv \neg q$.

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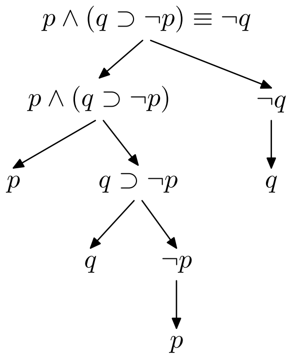
10. Direct subformulas of $p \wedge (q \supset \neg p) \equiv \neg q$.

$p \wedge (q \supset \neg p)$ and $\neg q$

11. Set of subformulas of $p \wedge (q \supset \neg p) \equiv \neg q$.

$$\{p \wedge (q \supset \neg p) \equiv \neg q, p \wedge (q \supset \neg p), \neg q, p, q \supset \neg p, q, \neg p\}$$

12. Construction tree of $p \wedge (q \supset \neg p) \equiv \neg q$.



13. Logical degree of $p \wedge q \supset (\neg p \equiv \neg q)$.

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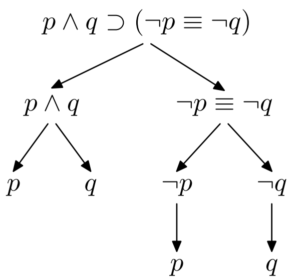
14. Direct subformulas of $p \wedge q \supset (\neg p \equiv \neg q)$.

$p \wedge q$ and $\neg p \equiv \neg q$

15. Set of subformulas of $p \wedge q \supset (\neg p \equiv \neg q)$.

$$\{p \wedge q \supset (\neg p \equiv \neg q), p \wedge q, \neg p \equiv \neg q, p, q, \neg p, \neg q\}$$

16. Construction tree of $p \wedge q \supset (\neg p \equiv \neg q)$.



17. Logical degree of $\neg p \wedge q \vee \neg q$.

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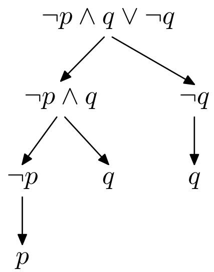
18. Direct subformulas of $\neg p \wedge q \vee \neg q$.

$\neg p \wedge q$ and $\neg q$

19. Set of subformulas of $\neg p \wedge q \vee \neg q$.

$$\{\neg p \wedge q \vee \neg q, \neg p \wedge q, \neg q, \neg p, q, \neg q, p\}$$

20. Construction tree of $\neg p \wedge q \vee \neg q$.



21. Logical degree of $\neg(p \wedge q \vee \neg q)$.

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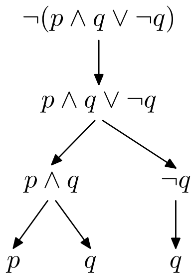
22. Direct subformulas of $\neg(p \wedge q \vee \neg q)$.

$p \wedge q \vee \neg q$

23. Set of subformulas of $\neg(p \wedge q \vee \neg q)$.

$$\{\neg(p \wedge q \vee \neg q), p \wedge q \vee \neg q, p \wedge q, \neg q, p, q\}$$

24. Construction tree of $\neg(p \wedge q \vee \neg q)$.



25. Logical degree of $p \supset \neg q \wedge \neg p$.

4

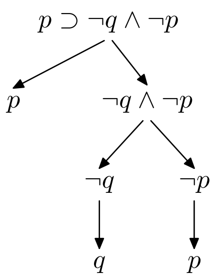
26. Direct subformulas of $p \supset \neg q \wedge \neg p$.

p and $\neg q \wedge \neg p$

27. Set of subformulas of $p \supset \neg q \wedge \neg p$.

$$\{p \supset \neg q \wedge \neg p, p, \neg q \wedge \neg p, \neg q, \neg p, q\}$$

28. Construction tree of $p \supset \neg q \wedge \neg p$.



29. Logical degree of $(p \supset \neg q) \wedge \neg p$.

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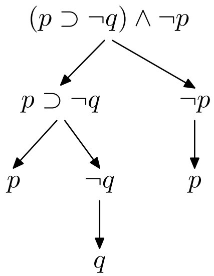
30. Direct subformulas of $(p \supset \neg q) \wedge \neg p$.

$(p \supset \neg q)$ and $\neg p$

31. Set of subformulas of $(p \supset \neg q) \wedge \neg p$.

$$\{(p \supset \neg q) \wedge \neg p, p \supset \neg q, \neg p, p, \neg q, q\}$$

32. Construction tree of $(p \supset \neg q) \wedge \neg p$.



33. Logical degree of $p \supset \neg q \equiv \neg p$.

4

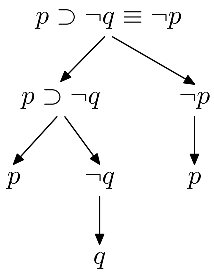
34. Direct subformulas of $p \supset \neg q \equiv \neg p$.

$p \supset \neg q$ and $\neg p$

35. Set of subformulas of $p \supset \neg q \equiv \neg p$.

$$\{p \supset \neg q \equiv \neg p, p \supset \neg q, \neg p, p, \neg q, q\}$$

36. Construction tree of $p \supset \neg q \equiv \neg p$.



37. Logical degree of $p \supset \neg(q \equiv \neg p)$.

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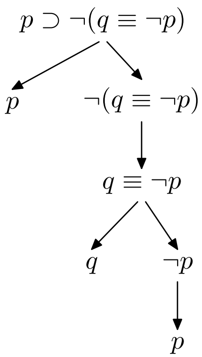
38. Direct subformulas of $p \supset \neg(q \equiv \neg p)$.

p and $\neg(q \equiv \neg p)$

39. Set of subformulas of $p \supset \neg(q \equiv \neg p)$.

$$\{p \supset \neg(q \equiv \neg p), p, \neg(q \equiv \neg p), q \equiv \neg p, q, \neg p\}$$

40. Construction tree of $p \supset \neg(q \equiv \neg p)$.



41. Logical degree of $\neg p \wedge \neg(q \vee \neg p)$.

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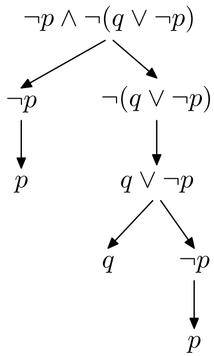
42. Direct subformulas of $\neg p \wedge \neg(q \vee \neg p)$.

$\neg p$ and $\neg(q \vee \neg p)$

43. Set of subformulas of $\neg p \wedge \neg(q \vee \neg p)$.

$$\{\neg p \wedge \neg(q \vee \neg p), \neg p, \neg(q \vee \neg p), p, q \vee \neg p, q, \neg p\}$$

44. Construction tree of $\neg p \wedge \neg(q \vee \neg p)$.



45. Logical degree of $\neg p \vee \neg(q \wedge \neg p)$.

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46. Direct subformulas of $\neg p \vee \neg(q \wedge \neg p)$.

$\neg p$ and $\neg(q \wedge \neg p)$

47. Set of subformulas of $\neg p \vee \neg(q \wedge \neg p)$.

$$\{\neg p \vee \neg(q \wedge \neg p), \neg p, \neg(q \wedge \neg p), q \wedge \neg p, q, p\}$$

48. Construction tree of $\neg p \vee \neg(q \wedge \neg p)$.