

Result:

NEPTUN ID: .....  Number of draft papers:

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**IMPORTANT!** The test contains 2 pages. The available time is 55 minutes. A successful exam will have at least 60 percent. Please indicate your NEPTUN ID on your draft pages too! You have to use blue ball pen to fill the test, the use of other tools is forbidden! The room is video-monitored.

## The definition of the $L_1$ first-order language

$$L_1 = \langle Con, \{x_1, x_2, \dots\}, \{c, f, g, p, q\}, Term, Form \rangle$$

- $c \in \mathcal{F}(0)$ ,
- $f \in \mathcal{F}(1)$ ,
- $g \in \mathcal{F}(2)$ ,
- $p \in \mathcal{P}(1)$ ,
- $q \in \mathcal{P}(2)$ ,

## Interpretation for the $L_1$ first-order language

$$\langle U, \varrho \rangle$$

- $U = \{10, 11, 12\}$ ,
- $\varrho(f) = f^I$ ,
- $\varrho(p) = p^I$ ,
- $\varrho(c) = 11$ ,
- $\varrho(g) = g^I$ ,
- $\varrho(q) = q^I$ ,

$$f^I(u) = 22 - u; \quad g^I(u, w) = \max(u, w)$$

$$p^I(u) = \begin{cases} 1 & \text{if } u = 11 \\ 0 & \text{otherwise} \end{cases}; \quad q^I(u, w) = \begin{cases} 1 & \text{if } u \neq w \\ 0 & \text{otherwise} \end{cases}$$

$$v(x_1) = 10; \quad v(x_2) = 12$$

## Exercise

1. Mark the appropriate box with an  $\mathcal{X}$ ! (5 point)

$c$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,	$\exists x_1 p(x_1)$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,
$q$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,	$\forall x_1 p(x_1)$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,
$\neg c$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,	$\neg p(g(x_1, x_2))$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,
$\neg p(x_1)$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,	$\neg q(x_1, x_2) \supset p(c)$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,
$g(f(x_1), c)$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,	$g(f(x_1), f(c))$	<input type="checkbox"/> $\in Form$ , <input type="checkbox"/> $\in Term$ , <input type="checkbox"/> $\notin L_1$ ,

2. Evaluate the formulas and terms above! (8 point)

3. Prove the sequene below, using natural deduction! (5 point)

$$((p \supset q) \wedge (p \supset \neg q)) \vdash \neg p$$

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