Questions for practical test

The practical test contains questions only from this document.
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- Zero-order syntax: formula, abbrevation, immediate subformula, set of subformula, logical degree, scope, main logical connective,
- Zero-order semantics: interpretation, formula valuation, tautology, contradiction, equivalance, logical consequence,
- First-order syntax: formula, term, abbrevation, immediate subformula, set of subformula, logical degree, scope, main logical connective, immediate subterm, set of subterms, functional degree, boundings, clean formula
- First-order semantics: interpretation, formula valuation, term valuation.

Zero-order syntax

Which one is a formula (but not an abbreviation)?

1. $\neg$
2. Y
3. $((X \wedge Y) \vee Z)$
4. $(X \wedge Y) \vee Z$
5. $\neg(X \vee Y$
6. $\neg X \vee Y))$
7. $(X \vee ZY)$
8. $(X \vee Z \wedge Y)$
9. $(\neg X)$
10. $\neg(X)$
11. $X \supset \neg(Y \wedge Z)$
12. $(X \supset \vee ZY)$
13. $((\neg X \supset Y) \supset \neg(X \vee Z))$
14. $(X \supset Z \supset Y)$
15. $\neg(X \vee Y \supset \neg\neg Z)$

Zero-order syntax

Put back the parentheses!

1. $\neg X \vee \neg\neg Y$
2. $Y \supset X \wedge \neg Z$
3. $\neg\neg X \wedge Y \supset Z$
4. $\neg(\neg X \wedge Y) \supset Z$
5. $X \wedge \neg Y \supset Z$
6. $X \supset \neg Y \wedge Z$
7. $X \supset (Y \wedge \neg X) \vee Y$
8. $\neg X \vee Y \supset \neg Y \wedge Z$

Remove as much patrentheses as possible from the following formulas!

1. $((X \wedge Y) \supset Z)$
2. $(X \wedge (Y \supset Z))$
3. $(\neg\neg X \wedge (Y \supset Z))$
4. $\neg\neg((X \wedge Y) \supset Z)$
5. $(\neg(X \wedge \neg Y) \supset (Z \vee \neg X))$
6. $(\neg(X \supset \neg Y) \wedge (Z \vee \neg X))$

Zero-order syntax

Determine the following properties!

(a) Logical degree,

(b) immediate subformulas,

(c) set of subformulsa,

(d) main logical connective.

1. X
2. $\neg\neg X$
3. $X \vee X$
4. $\neg X \vee Y$
5. $X \vee Y \supset Z$
6. $\neg(X \vee Y)$
7. $(\neg(X \wedge \neg Y) \supset (Z \vee \neg X))$
8. $\neg(X \supset Y \vee \neg Z) \wedge \neg Y \supset Z$
9. $\neg(\neg\neg X \supset Y \vee \neg\neg Z)$
10. $\neg X \wedge X \supset Y$
11. $(\neg\neg X \wedge (Y \supset Z))$
12. $\neg\neg((X \wedge Y) \supset Z)$

Zero-order semantics

Evaluate the expressions!

(a) $\mathcal{I}_1(X) = \mathit{true}, \quad \mathcal{I}_1(Y) = \mathit{false}, \quad \mathcal{I}_1(Z) = \mathit{false}, \quad \mathcal{I}_1(W) = \mathit{false}$

1. $|\neg(X \wedge Z) \supset Y|^{\mathcal{I}_1}$
2. $|(X \supset \neg Y) \supset \neg(Y \supset Z)|^{\mathcal{I}_1}$
3. $|(X \vee \neg Y) \wedge \neg(Y \supset Z)|^{\mathcal{I}_1}$
4. $|(X \vee Y \supset \neg Z) \supset \neg Y \wedge Z|^{\mathcal{I}_1}$
5. $|(X \supset \neg Y) \supset \neg(Y \supset Z)|^{\mathcal{I}_1}$
6. $|\neg(\neg(X \supset Y \vee Z) \wedge W \supset Z)|^{\mathcal{I}_1}$
7. $|\neg(X \wedge \neg Y) \supset Z \vee \neg X|^{\mathcal{I}_1}$
8. $|\neg X \wedge Z \supset Y \vee \neg W|^{\mathcal{I}_1}$
9. $|(\neg X \wedge Z \supset Y) \vee \neg W|^{\mathcal{I}_1}$
10. $|\neg\neg((X \wedge Y) \supset Z)|^{\mathcal{I}_1}$

Zero-order semantics

Evaluate the expressions!

(b) $\mathcal{I}_2(X) = \mathit{false}, \quad \mathcal{I}_2(Y) = \mathit{false}, \quad \mathcal{I}_2(Z) = \mathit{true}, \quad \mathcal{I}_2(W) = \mathit{false}$

1. $|\neg (X \wedge Z) \supset Y|^{\mathcal{I}_2}$
2. $|(X \supset \neg Y) \supset \neg (Y \supset Z)|^{\mathcal{I}_2}$
3. $|(X \vee \neg Y) \wedge \neg (Y \supset Z)|^{\mathcal{I}_2}$
4. $|(X \vee Y \supset \neg Z) \supset \neg Y \wedge Z|^{\mathcal{I}_2}$
5. $|(X \supset \neg Y) \supset \neg (Y \supset Z)|^{\mathcal{I}_2}$
6. $|\neg(\neg(X \supset Y \vee Z) \wedge W \supset Z)|^{\mathcal{I}_2}$
7. $|\neg(X \wedge \neg Y) \supset Z \vee \neg X|^{\mathcal{I}_2}$
8. $|\neg X \wedge Z \supset Y \vee \neg W|^{\mathcal{I}_2}$
9. $|(\neg X \wedge Z \supset Y) \vee \neg W|^{\mathcal{I}_2}$
10. $|\neg\neg((X \wedge Y) \supset Z)|^{\mathcal{I}_2}$

Zero-order semantics

Construct an interpretation (if possible) which satisfies the followings!

1. $|\neg(Y \vee \neg Z \supset X \vee Z)|^{\mathcal{I}_1} = true$
2. $|X \supset Y \vee Z|^{\mathcal{I}_2} = true; \quad |\neg(\neg X \vee Y)|^{\mathcal{I}_2} = true; \quad |Z \supset U \vee Y|^{\mathcal{I}_2} = true$
3. $|\neg(X \wedge Y \supset Z)|^{\mathcal{I}_3} = true; \quad |\neg U \supset \neg Z|^{\mathcal{I}_3} = true; \quad |Y \supset \neg U|^{\mathcal{I}_3} = true$

Proove or disprove that the following formulas are tautologies or contradictions!

1. $\neg X \wedge \neg Y \supset (X \vee Y)$
2. $\neg(X \supset Y) \supset \neg(X \wedge \neg Y)$
3. $\neg(X \supset \neg Y) \wedge (\neg Z \supset Y)$
4. $\neg(X \wedge \neg(Y \wedge \neg(Z \vee \neg X)))$
5. $\neg X \supset (Y \supset (Z \supset \neg X))$
6. $X \supset (Y \supset (Z \supset \neg X))$

Zero-order semantics

Proove or disprove, that the following formulas are logically equivalent!

1. $(X \vee Y) \wedge X \quad \overset{?}{\sim}_0 \quad X$
2. $(X \wedge Y) \vee X \quad \overset{?}{\sim}_0 \quad X$
3. $X \vee Y \supset Z \quad \overset{?}{\sim}_0 \quad (\neg X \wedge \neg Y) \vee Z$
4. $(X \supset Y) \supset Z \quad \overset{?}{\sim}_0 \quad (Y \supset X) \supset Z$
5. $X \supset (Y \supset Z) \quad \overset{?}{\sim}_0 \quad X \supset (Z \supset Y)$
6. $X \supset (Y \supset Z) \quad \overset{?}{\sim}_0 \quad (X \supset Y) \supset Z$
7. $X \vee Y \supset Z \quad \overset{?}{\sim}_0 \quad Z \vee Y \supset X$
8. $(X \supset Y) \vee (Z \supset W) \quad \overset{?}{\sim}_0 \quad \neg(X \wedge Z) \vee \neg(\neg Y \wedge \neg W)$

Zero-order semantics

1. Proove or disprove, that the $K \supset X$ formula is a logical consequence of the formulas below!
$$(X \vee \neg Y) \wedge (Z \supset W)$$
$$\neg(Y \vee Z \supset X \vee U)$$

2. Proove or disprove, that the $\neg W \supset Z$ formula is a logical consequence of the formulas below!
$$(X \vee \neg Y) \wedge (Z \supset W)$$
$$\neg(Y \vee Z \supset X \vee U)$$

First order syntax

Underline the π_1 type terms of the L_1 language!

$$L_1 = \langle \{\pi_1\}, \{P, Q, R\}, \{f, g\}, \{c\} \rangle$$

- $x, y, z, \ldots$ are π_1 type variables
- $\nu_1(P) = (\pi_1), \quad \nu_1(Q) = (\pi_1, \pi_1), \quad \nu_1(R) = (\pi_1, \pi_1)$
- $\nu_2(f) = (\pi_1, \pi_1), \quad \nu_2(g) = (\pi_1, \pi_1)$
- $\nu_3(c) = (\pi_1)$

1. c
2. x
3. $F(x)$
4. $f(c, x)$
5. $c(x)$
6. $f(c)$
7. $f(g(x))$
8. $f(g(x, y), z)$
9. $P(x)$
10. $Q(x)$
11. $\neg f(c)$
12. $g(c, f(x))$

First order syntax

Underline the formulas of the L_1 language!

$$L_1 = \langle \{\pi_1\}, \{P, Q, R\}, \{f, g\}, \{c\} \rangle$$

- $x, y, z, \ldots$ are π_1 type variables
- $\nu_1(P) = (\pi_1), \quad \nu_1(Q) = (\pi_1, \pi_1), \quad \nu_1(R) = (\pi_1, \pi_1)$
- $\nu_2(f) = (\pi_1, \pi_1), \quad \nu_2(g) = (\pi_1, \pi_1)$
- $\nu_3(c) = (\pi_1)$

1. $P(c)$
2. $Q(x)$
3. $\neg P(\neg f(x))$
4. $R(f(x), g(x))$
5. $\exists x f(x)$
6. $\forall \neg P(x) \wedge f(x)$
7. $\exists c R(f(c), y)$
8. $\neg \exists x R(P(c), f(x))$
9. $P(x) \supset \exists x Q(x, x)$
10. $\exists f(x) P(f(x))$
11. $\neg f(c) \wedge \forall y Q(y, y)$
12. $P(x \wedge y)$

First order syntax

Let $P, Q, R, \ldots$ predicate symbol, $f, g, h, \ldots$ function symbol, $a, b, c, \ldots$ constant symbol and $x, y, z, \ldots$ variables of a first order language!

- Specify the functional degree of the terms and logical degrees of the formulas,
- specify the immediate sub expressions!

1. $P(x) \vee Q(x, y) \supset P(f(x)) \wedge Q(f(x), y)$
2. $\forall x(P(x) \vee Q(x, y) \supset P(x) \wedge Q(x, y))$
3. $f(g(x, y))$
4. $g(f(x), f(x))$
5. $\neg P(g(f(x), f(x)))$

First order syntax

Clean the formulas!

1. $\forall x \forall y (Q(x, y) \supset \neg Q(y, x))$
2. $\forall x (P(x) \vee Q(x, c)) \supset \exists x \neg Q(x, c)$
3. $\forall x \forall y (P(x) \wedge Q(y, x) \supset \neg P(y))$
4. $Q(x, f(x)) \wedge \neg Q(y, f(y))$
5. $\forall x \exists y Q(y, x) \wedge \neg \exists y \forall x Q(y, x)$
6. $\neg \forall x \forall y (Q(x, y) \supset \neg Q(y, x))$
7. $\neg(\forall x (P(x) \vee Q(x, c)) \supset \exists x \neg Q(x, c))$
8. $\exists x \exists y \neg (P(x) \wedge Q(y, x) \supset \neg P(y))$
9. $\forall x \exists y Q(y, x) \vee \neg \exists y \forall x Q(y, x)$

First-order semantics

Evaluate the expression: $|\forall x(Q(f(x), y) \supset \neg Q(x, f(y)))|^{I,\kappa}$

$$L = \langle \{\pi\}, \{P, Q\}, \{f, g\}, \{c\} \rangle$$

- $x, y, z, \ldots$ are π type variables
- $\nu_1(P) = (\pi)$, $\nu_1(Q) = (\pi, \pi)$
- $\nu_2(f) = (\pi, \pi)$, $\nu_2(g) = (\pi, \pi, \pi)$
- $\nu_3(c) = (\pi)$

$$I = \langle I_{Srt}, I_{Pr}, I_{Fn}, I_{Cnst} \rangle$$

- $I_{Srt}(\pi) = \{1, 2, 3, 4\}$
- $I_{Pr}(P) = P^I$, $I_{Pr}(Q) = Q^I$
- $I_{Fn}(f) = f^I$, $I_{Fn}(g) = g^I$
- $I_{Cnst}(c) = 2$

$$\begin{aligned} f^I(\alpha) &= 5 - \alpha \\ g^I(\alpha, \beta) &= |\alpha - \beta| + 1 \\ P^I(\alpha) &= \begin{cases} true & \text{if } \alpha = 1 \text{ or } \alpha = 4 \\ false & \text{othervise} \end{cases} \\ Q^I(\alpha, \beta) &= \begin{cases} true & \text{if } \alpha > \beta \\ false & \text{othervise} \end{cases} \end{aligned}$$

$$\kappa(x) = 1, \quad \kappa(y) = 3$$

First-order semantics

Evaluate the expression: $|\exists y(Q(f(c), y) \supset \neg Q(c, f(y)))|^{I,\kappa}$

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First-order semantics

Evaluate the expression: $|\exists x P(x) \supset \forall x \neg Q(x, f(x))|^{I,\kappa}$

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First-order semantics

Evaluate the expression: $|\forall x \exists y \neg Q(x, f(y))|^{I,\kappa}$

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$$\kappa(x) = 1, \quad \kappa(y) = 3$$

First-order semantics

Evaluate the expression: $|\forall x \forall y (Q(x, f(y)) \vee Q(f(y), x))|^{I,\kappa}$

$$L = \langle \{\pi\}, \{P, Q\}, \{f, g\}, \{c\} \rangle$$

- $x, y, z, \ldots$ are π type variables
- $\nu_1(P) = (\pi)$, $\nu_1(Q) = (\pi, \pi)$
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