**Basic concepts of**

# Set theory

Logic in computer science

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# Basic concepts of set theory

A **set** is a collection of objects which are called the members or elements of that set. We can say that a memeber of a set is in the set (or belongs to the set), and the other object are not in the set.
We can define a set:

- by listing all its members:
  Let $H_1$ be a set of students: $H_1 = \{Joe, Peter, Amy\}$
  Let $H_2$ be a set of octal digits: $H_2 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ or $H_2 = \{0, 1, \ldots, 7\}$
- or with relevant properties:
  Let $H_3$ be a set of students in the room:
  $H_3 = \{h | \text{where } h \text{ is a student in the room}\}$
- we denote the empty set with the following symbol: $\varnothing$

# Basic concepts of set theory

Notations:

- $h \in H$ denotes that $h$ belongs to $H$ (or $h$ is a member of $H$)
  $5 \in \{0, 1, 2, 3, 4, 5, 6, 7\}$
- $h \notin H$ denotes that $h$ does not belong to $H$ (or $h$ is not a member of $H$)
  $Thomas \notin \{Joe, Peter, Amy\}$

# Basic concepts of set theory

- A set $H_1$ is a subset of a set $H_2$ if and only if, every element of $H_1$ is also an element of $H_2$. Such a relation between sets is denoted by $H_1 \subseteq H_2$.
- When $H_1 \subseteq H_2$ and $H_2 \subseteq H_1$ then, $H_1$ and $H_2$ has the same members. Notation: $H_1 = H_2$.
- Two sets are said to be disjoint if they have no element in common.
- The set of all subsets of a set $H$ is called the **power set** of $H$ and denoted as $\mathcal{P}(H)$
  Example: $\mathcal{P}(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

# Basic operations

Basic operations over $\mathcal{P}(U)$:
Let $H_1$ and $H_2$ be sets form $\mathcal{P}(U)$!

- union: $H_1 \cup H_2 \rightleftharpoons \{h|h \in H_1 \text{ or } h \in H_2\}$.
- intersection: $H_1 \cap \ H_2 \rightleftharpoons \{h|h \in H_1 \text{ and } h \in H_2\}$.
- difference: $H_1 \setminus H_2 \rightleftharpoons \{h|h \in H_1 \text{ and } h \notin H_2\}$
- complement: $\overline{H_1} \rightleftharpoons \{h|h \in U \text{ and } h \notin H_1\}$
- Cartesian product (or product set): Let $H_1, H_2, \ldots, H_n$ $(n > 0)$ be non-empty sets:
  $H_1 \times H_2 \times \ldots \times H_n \rightleftharpoons$
  $\{(h_1, h_2, \ldots, h_n)|h_1 \in H_1 \text{ and } h_2 \in H_2 \text{ and } \ldots \text{ and } h_n \in H_n)\}$

# Binary relations

With a binary relation, we will define relations between elements of two sets. When $(h_1, h_2) \in R$ ( $h_1$ is related to $h_2$) then we use the notation: $h_1 R h_2$.
The $R \subseteq H \times H$ is a binary relation on $H$.
A binary relation is

- a reflexive relation, if $hRh$ for all $h \in H$,
- an anti-reflexive relation, if there is no $h \in H$ where $hRh$,
- a symmetric relation, if (for all $h_1, h_2 \in H$) when $h_1 R h_2$ then $h_2 R h_1$,
- an anti-symmetric relation, if there is no $h_1, h_2 \in H$ where $h_1 R h_2$ and $h_2 R h_1$,
- a transitive relation, if (for all $h_1, h_2, h_3 \in H$) when $h_1 R h_2$ and $h_2 R h_3$ then $h_1 R h_3$

# Domain and range

Let $R$ be a binary relation on $H$! ($R \subseteq H \times H$)

- The domain of $R$ (denoted by $\mathrm{Dom}(R)$) is the following set:

$$\{h_1 | (h_1, h_2) \in R \text{ for some } h_2\}$$

- The range of $R$ (denoted by $\mathrm{Rng}(R)$) is the following set:

$$\{h_2 | (h_1, h_2) \in R \text{ for some } h_1\}$$

# Functions

Let $R$ be a binary relation on $H$! ($R \subseteq H \times H$)

- $R$ is called injective (left unique) relation if, for all $x \in H$ and $y \in H$:

$$xRz \quad \text{and} \quad yRz \quad \Longrightarrow \quad x = y$$

- $R$ is called surjective if, for all $y \in H$ there exists an $x \in H$ where $xRy$. It's also called right total because $\mathrm{Rng}(R) = H$.
- $R$ is called bijective when, it is both injective and surjective.
- $R$ is called functional (right unique) or partial function if, for all $x, y, z \in H$:

$$xRy \quad \text{and} \quad xRz \quad \Longrightarrow \quad y = z$$

- $R$ is called function if, it is functional and $\mathrm{Dom}(R) = H$. In other words: a function is functional and left total relation.