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Plenary sessions

Strong approximation, Skorohod representation, St. Petersburg paradox: some forgotten results of Paul Lévy

I. Berkes

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and
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In this lecture, we discuss some remarks in Paul Lévy's paper "*Propriétés asymptotiques des sommes de variables aléatoires indépendantes ou enchainées*", *J. Math. Pures Appl.* 14 (1935), 347–402. Lévy's remarks have apparently been forgotten in the shadow of the celebrated results of his paper, even though they contain important results and ideas of probability theory discovered decades later. Among others, Lévy proves a remarkable limit theorem for i.i.d. sums with irregularly varying tails, containing the striking limit theorem for St. Petersburg sums used to clarify the St. Petersburg paradox 50 years later. The proof uses a coupling argument, similar to Skorohod representation, and provides a strong (pointwise) approximation result, 30 years before Strassen's theory. Lévy was also the first to use the quantile transform, the basic tool leading to the deep approximation results in the years 1970-1990 and utilized it, among others, to prove a pointwise decomposition of stable variables, another celebrated result from the 1980's. In addition to discussing Lévy's arguments, we also formulate some new results using his ideas.

Dimension reduction for functional data

Siegfried Hörmann

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Data in many fields of science are sampled from processes that can most naturally be described as functional. Examples include growth curves, temperature curves, curves of financial transaction data and patterns of pollution data. Functional data analysis (FDA) is concerned with the statistical analysis of such data.

An important tool in many empirical and theoretical problems related to FDA is the *functional principal analysis (FPCA)* which allows to represent or approximate curves in low dimension. It is certainly the most common approach to obtain dimension reduction for functional data. In fact, it achieves in some sense optimal dimension reduction if data are independent. However, it is all but uncommon that functional data are serially correlated. A typical example is if the observations are segments from a continuous time process (e.g. days). Then, although cross-sectionally uncorrelated for a fixed observation, the classical FPC-score vectors have non-diagonal cross-correlations. This means that we cannot analyze them componentwise (like in the i.i.d. case), but we need to consider them as vector time series which are less easy to handle and interpret. In this talk we will present a time series version of functional PCA. The idea is to transform the (possibly infinite dimensional) functional time series, into a vector time series of low dimension (in practice 3 or 4, say), where the individual component processes are mutually uncorrelated, and explain a bigger part of the dynamics and variability of the original process. We demonstrate the method on real data and discuss potential applications.

The talk is based on joint work with Łukasz Kidziński (Stanford) and Marc Hallin (ULB).

GG-mixed Poisson distributions

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A wide and flexible family of discrete distributions is considered, the so-called GG-mixed Poisson distributions that are mixed Poisson distributions in which the mixing laws belong to the class of generalized gamma distributions (GG-distributions). The latter contains practically all most popular absolutely continuous distributions concentrated on the non-negative half-line. The GG-mixed Poisson distributions seem to be very promising in the statistical description of many real phenomena being almost universal models for the description of statistical regularities in discrete data.

Analytic properties of GG-mixed Poisson distributions are studied. A GG-distribution is proved to be a mixed exponential distribution if and only if the shape and exponent power parameters are no greater than one. The mixing distribution is written out explicitly as a scale mixture of strictly stable laws concentrated on the nonnegative halfline. As a corollary, the representation is obtained for the GG-mixed Poisson distribution as a mixed geometric distribution. The corresponding scheme of Bernoulli trials with random probability of success is considered. Within this scheme, a random analog of the Poisson theorem is proved establishing the convergence of mixed binomial distributions to mixed Poisson laws. Limit theorems are proved for random sums of independent random variables in which the number of summands has the GG-mixed Poisson distribution and the summands have both light- and heavy-tailed distributions. The class of limit laws is wide enough and includes the so-called generalized variance gamma distributions. Various representations for the limit laws are obtained in terms of mixtures of Mittag-Leffler, Linnik or Laplace distributions. Limit theorems are proved establishing the convergence of the distributions of statistics constructed from samples with random sizes obeying the GG-mixed Poisson distribution to generalized variance gamma distributions. Some applications of GG-mixed Poisson distributions in meteorology are discussed.

The eigenstructure of sample covariance matrices for high-dimensional heavy-tailed stochastic volatility models

Thomas Mikosch

University of Copenhagen, Denmark

This is joint work with Johannes Heiny (Aarhus University).

We are interested in the asymptotic behavior of the eigenvalues of the sample covariance matrix $\mathbf{X}\mathbf{X}'$ where the data matrix $\mathbf{X} = \mathbf{X}_n = (X_{it})_{i=1,\dots,p;t=1,\dots,n}$ has the structure of a stochastic volatility model, i.e., $X_{it} = \sigma_{it}Z_{it}$ and the stationary volatility field (σ_{it}) is independent of the iid field (Z_{it}) . We assume that Z_{it} has regularly varying tails with index $\alpha \in (0, 4)$ and that σ_{it} has lighter tails. We are interested in the case when the dimension $p = p_n \rightarrow \infty$. Then the eigenvalues of $\mathbf{X}\mathbf{X}'$ are essentially determined by the diagonal elements $\mathbf{X}_{ii}$, $i = 1, \dots, n$. We consider limit theory of Poisson-type for these eigenvalues and also discuss the structure of the corresponding eigenvectors.

Central limit theorem for random walk in doubly stochastic random environment

Bálint Tóth

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It is proved that an infrared bound on the correlations of the local drift field, called the H_{-1} -condition, is sufficient for the central limit theorem with diffusive scaling to hold for the random walks in doubly stochastic random environments. This result settles a notorious issue in the context of random walks in random environments. The talk will be based partly on joint work with Gady Kozma (Weizmann Institute).

The Mittag-Leffler functions, related distributions, processes and limit theorems

V. Yu. Korolev and V. V. Uchaikin

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The Mittag-Leffler (M-L) functions can be considered as a generalization of the exponential function whose role in theoretical physics can hardly be overestimated. Nevertheless, only the last two decades or so have been marked by increased attention to these functions [1]. This review gives a short description of their main properties, containing both known results related to fractional differential equations and original results concerning the properties of the M-L probability distribution such as its geometric stability, various mixture representations and relations with the Linnik distributions. Limit theorems for statistics constructed from samples with random sizes (including continuous-time random walks and sums of a random number of independent random variables *with finite variances*) are presented establishing the convergence of the distributions of such statistics to these distributions.

The applications of the M-L functions to random processes are considered including fractional Poisson process, time-fractional Boltzmann process, bifractional generalization of the Brownian diffusion process and some others. It is shown that long-time asymptotics of such processes generates fractional stable distributions and fractional Poisson limit distributions.

Some examples of application of these models to description of processes in cosmic scales are demonstrated: cosmic ray transport, magnetic field turbulence, pulsar signal propagation, image processing in astronomy.

This work is partially supported by Russian Foundation for Basic Research (projects 15-07-04040 and 16-01-00556).

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Organized and contributed sessions

M-estimation of unknown parameter in competing risks model under random censoring from both sides

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In competing risks model (see [1]), we are interested in observing of random variable (r.v.) Z with distribution function (d.f.) H and pairwise disjoint events $\{A^{(i)}, i \in J = \{1, \dots, k\}\}$. In fact, we interested in joint properties of pairs $\{(Z, A^{(i)}), i \in J\}$. Let's introduce subdistribution functions $\{H(x; i) = P(Z \leq x, A^{(i)}), (x; i) \in R \times J\}$, for which $H(x; 1) + \dots + H(x; k) = H(x)$. Here we suppose that the pairs $\{(Z, A^{(i)}), i \in J\}$ are censored from right and left by r.v.-s Y and L with corresponding d.f.-s G and K . Suppose that r.v.-s $\{Z, Y, L\}$ are independent. Observation is available the sample $S^{(n)} = \left\{ \left(\zeta_j; \chi_j^{(0)}; \chi_j^{(1)}, \dots, \chi_j^{(k)} \right), j = 1, \dots, n \right\}$, where $\zeta_j = \max\{L_j, \min\{Z_j, Y_j\}\}$, $\chi_j^{(i)} = I(D_j^{(i)})$, $i \in \bar{J} = J \cup \{0\}$, $D_j^{(0)} = \{\min\{Z_j, Y_j\} < L_j\} \cup \{L_j \leq Y_j < Z_j\}$, $D_j^{(i)} = A_j^{(i)} \cap \{L_j \leq Z_j \leq Y_j\}$, $i \in J$ and $\{Z_j, L_j, D_j^{(0)}, D_j^{(1)}, \dots, D_j^{(k)}, j \geq 1\}$ consequence of independent and identically distributed copies of aggregate $\{Z, L, Y, D^{(0)}, D^{(1)}, \dots, D^{(k)}\}$. It is not difficult to see that d.f. of r.v. $\zeta = \max\{L, \min\{Z, Y\}\}$ is $E(x) = P(\zeta \leq x) = K(x)(1 - (1 - G(x))(1 - H(x)))$. Note that in the sample $S^{(n)}$ random pairs $(Z_j, A_j^{(i)})$ observable only in the case of $\chi_j^{(i)} = 1, i \in J$. Consider survival functionals $1 - F(x; i) = \exp(-\Lambda_\tau(x; i))$, $i \in J$ and their estimators $1 - F_{n\tau}(x; i) = \exp(-\Lambda_{n\tau}(x; i))$, $i \in J$, where $\Lambda_\tau(x; i) = \int_{[\tau; x]} \frac{dH(u; i)}{1 - H(u-)}, (x; i) \in [\tau, \infty) \times J$, $\Lambda_{n\tau}(x; i) = \frac{1}{n} \sum_{j=1}^n \frac{I(\zeta_j \leq x, \chi_j^{(i)} = 1)}{q_n(\zeta_j)}$, $q_n(x) = K_n(x) - \Delta T_{1n}(x; 0)$, $T_{1n}(x; 0) = \frac{1}{n} \sum_{j=1}^n I(\zeta_j \leq x, \chi_j^{(0)} = 1)$, $E_n(x) = \frac{1}{n} \sum_{j=1}^n I(\zeta_j \leq x)$, $K_n(x) = \exp\left(-\int_{[x, \infty)} \frac{dT_{1n}(u; 0)}{E_n(u-)}\right)$.

Let $\mathcal{L}_q(Q)$ be the space of functions $f : \mathbb{R} \rightarrow \mathbb{R}$ with the norm $\|f\|_{Q, q} = \left\{ \int_{\mathbb{R}} |f|^q dQ \right\}^{1/q}$, where $Q(x) = \int_{\tau}^x \frac{dE(u)}{[K(u)(1 - \gamma(u-))]^2}$.

Let $h_q(\varepsilon) = \log N_{[]}(\varepsilon, \mathcal{F}, \mathcal{L}_q(Q))$ be the metric entropy with the bracketing number $N_{[]}(\varepsilon, \mathcal{F}, \mathcal{L}_q(Q))$ of the class $\mathcal{F}$ in $\mathcal{L}_q(Q)$. We define also the integral of the metric entropy with bracketing as $J_{[]}^{(q)}(\delta) = J_{[]}(\delta; \mathcal{F}; \mathcal{L}_q(Q)) = \int_0^\delta (h_q(\varepsilon))^{1/2} d\varepsilon$, $0 < \delta \leq 1$. Introduce $\mathcal{F}$ -indexed process for each $i \in J$ as $G_n^{(i)} f =$

$\int_{\tau}^T f(x) d(F_{n\tau}(x; i) - F_{\tau}(x; i)), f \in \mathcal{F}$, where $\tau < T < T_Q = \sup [x : Q(x) < \infty]$.

Theorem 1. Suppose that $\mathcal{F} \subset \mathcal{L}_1(Q)$ and $J_{\square}^{(1)}(1) < \infty$. Then as $n \rightarrow \infty$

$$\sup_{f \in \mathcal{F}} \left| G_n^{(i)} f \right| \xrightarrow{a.s.} 0, \quad i \in J.$$

Theorem 2. Let class $\mathcal{F}$ such that $\mathcal{F} \subset \mathcal{L}_2(E)$ and $J_{\square}^{(2)}(1) < \infty$. Then for each $i \in J$ as $n \rightarrow \infty$ processes $\left\{ \sqrt{n} G_n^{(i)} f, f \in \mathcal{F} \right\}$ converges weakly in $l^{\infty}(\mathcal{F})$ to corresponding mean zero Gaussian process.

These theorems can be used for estimation of unknown parameter $\theta \in \Theta \subseteq \mathbb{R}$. Let $f_{\theta} : \mathbb{R} \rightarrow \mathbb{R}$ be some loss function and $\mathcal{F} = \{f_{\theta}, \theta \in \Theta\}$. For example, a) In location estimation: $\Theta = \mathbb{R}$ and $f_{\theta}(x) = (x - \theta)^2$ (estimating the mean); $f_{\theta}(x) = |x - \theta|^2$ (estimating the median), b) In maximum likelihood: $\{h_{\theta}, \theta \in \Theta\}$ - family of densities and $f_{\theta}(x) = -\log h_{\theta}(x)$. We estimate θ by M - estimator

$$\theta_n = \text{Arg} \max_{(\theta; i) \in \Theta \times J} \left\{ \int f_{\theta}(x) dF_{n\tau}(x; i) \right\}.$$

Assume that θ_n exists. Then under mild conditions on class $\mathcal{F}$, from theorems 1 and 2 one can obtain a strong consistency and asymptotical normality properties of estimator θ_n .

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Semi-analytical option pricing under hybrid double Heston-CIR jump diffusion

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This paper discusses an extension of the jump-diffusion version of the Double Heston stochastic volatility model for the underlying price process to provide a more flexible model for the term structure of volatility. The jumps in stock price and in one of the variance processes occur concurrently and their magnitudes are correlated, the second variance process is of the standard Heston dynamics. We assume, in addition, that the stochastic interest rate is governed by the CIR dynamics. The instantaneous volatilities are correlated with the dynamics of the stock price process, whereas the short-term rate is assumed to be independent of the dynamics of the price process and its volatility. The main result furnishes a semi-analytical formula for the price of the European call option in the hybrid call option/interest rates model. Numerical results show that the model implied volatilities are comparable for in-sample but outperform out-of-sample implied volatilities compared to the benchmark Heston model and Double Heston volatility model for calls on the *S&P 500* index.

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On the limit of dense preferential attachment random graphs

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In the last decade, the theory of graph limits (combined with tools from probability theory) has been used to solve various problems on the asymptotic behavior of random graphs [3]. In the current work, we give a kind of an upper bound on the speed of convergence for a sequence of convergent random graphs.

In general, the family of preferential attachment graphs has been widely studied after the seminal work of Albert and Barabási [1]. In these models, new vertices are added to the graph step by step, and old vertices of larger degree have more chance to be connected to the new ones. Although many variants of preferential attachment models have been examined, most of them are sparse in the sense that the number of edges is linear as a function of the number of vertices.

In the current work, we investigated a dense version of the preferential random graph, where the number of edges is constant times n^2 (where n is the number of vertices). In our case, a Pólya urn process with n urns is used to construct the graph. We proved that the expectation of the distance of the preferential attachment graph and a randomized version of its limit is at most $O(\log^2 n \cdot n^{-1/3})$, in an appropriate distance (the so-called jumble norm). The model, the main results of [1] and these notions of graph convergence and graph distances would be presented in the talk.

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Combining predictive distributions for calibration of ensemble forecasts for wind speed

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Statistical post-processing techniques are now widely used to correct systematic biases and errors in calibration of ensemble forecasts obtained from multiple runs of numerical weather prediction models. A standard approach is the ensemble model output statistics (EMOS) method, where the forecast distribution is given by a single parametric law with parameters depending on the ensemble members. Choosing an appropriate parametric family for the weather variable of interest is a critical, however, often non-trivial task, and has been the focus of much recent research. We assess the merits of combining predictive distributions from truncated normal [1] and log-normal [2] EMOS models for wind speed using the state of the art forecast combination methods. In two case studies with wind speed forecasts from two ensemble prediction systems we investigate the performance of the (spread-adjusted) linear pool, the beta transformed linear pool, and a recently introduced Bayesian calibration approach [3] and we also propose a computationally efficient 'plug-in' approach to determining combination weights in the linear pool that is specific to post-processing applications.

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Asymptotic properties of MLE for the growth rate of an α -stable CIR process

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We consider an α -stable Cox–Ingersoll–Ross (CIR) process given by the SDE

$$dY_t = (a - bY_t) dt + \sigma\sqrt{Y_t} dW_t + \delta \sqrt[\alpha]{Y_t} dL_t, \quad t \in [0, \infty),$$

with a deterministic initial value $y_0 \in [0, \infty)$, where $a \in [0, \infty)$, $b \in (-\infty, \infty)$, $\sigma, \delta \in (0, \infty)$, $\alpha \in (1, 2)$, $(W_t)_{t \in [0, \infty)}$ is a 1-dimensional standard Wiener process, and $(L_t)_{t \in [0, \infty)}$ is a spectrally positive strictly α -stable Lévy process with characteristic function

$$\mathbb{E}(e^{i\theta L_1}) = \exp \left\{ \int_0^\infty (e^{i\theta z} - 1 - i\theta z) C_\alpha z^{-1-\alpha} dz \right\}, \quad \theta \in (-\infty, \infty),$$

where $C_\alpha := (\alpha\Gamma(-\alpha))^{-1}$ and Γ denotes the Gamma function. We suppose that $(W_t)_{t \in [0, \infty)}$ and $(L_t)_{t \in [0, \infty)}$ are independent.

Supposing that a , σ , δ and α are known, we study asymptotic properties of the maximum likelihood estimator (MLE) for the growth rate b of the model based on continuous time observations $(Y_t)_{t \in [0, T]}$ as $T \rightarrow \infty$. If $a \in (0, \infty)$ or $y_0 \in (0, \infty)$, then for each $T \in (0, \infty)$, we show that there exists a unique MLE $\hat{b}_T$ of b almost surely having the form

$$\hat{b}_T = - \frac{Y_T - y_0 - aT - \delta \int_0^T \sqrt[\alpha]{Y_u} dL_u}{\int_0^T Y_s ds},$$

provided that $\int_0^T Y_s ds$ is positive (which holds almost surely).

We distinguish three cases: subcritical, critical and supercritical cases according to $b > 0$, $b = 0$ and $b < 0$. In all cases, somewhat surprisingly, we prove strong consistency of the MLE in question. In the subcritical case we prove asymptotic normality. In the supercritical case asymptotic mixed normality is shown, although the limiting mixed normal (but non-normal) distribution is given in a complicated way. In the critical case the description of the asymptotic behaviour of the MLE is still an open question, even the scaling factor is unclear.

On regularly varying sequences and arrays

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We consider weakly dependent stationary and regularly varying stochastic processes in discrete time and study their long term behavior. After describing the limiting distribution of individual clusters of extremes, we show a new type of point process convergence theorem. Using it, we are able to present a new functional limit theorem, which covers several time series models relevant in applications, for which standard limiting theory in the space D of càdlàg functions fails. To describe the limit of partial sums in this more general setting we use the concept of so-called decorated càdlàg functions. The method can be extended to analyze asymptotics of records and records times for stationary sequences and arrays.

Based on a joint work with Hrvoje Planinić (University of Zagreb) and Philippe Soulier (Université Paris Ouest Nanterre).

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Asymptotic properties of maximum likelihood estimator for the growth rate for a jump-type CIR process based on continuous time observations

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We consider a jump-type Cox–Ingersoll–Ross (CIR) process

$$dY_t = (a - bY_t) dt + \sigma\sqrt{Y_t} dW_t + dJ_t, \quad t \in [0, \infty),$$

with a deterministic initial value $y_0 \in [0, \infty)$, where $a \in [0, \infty)$, $b \in (-\infty, \infty)$, $\sigma \in (0, \infty)$, $(W_t)_{t \in [0, \infty)}$ is a 1-dimensional standard Wiener process, and $(J_t)_{t \in [0, \infty)}$ is an independent subordinator (an increasing Lévy process) with zero drift and with Lévy measure m concentrating on $(0, \infty)$ such that $\int_0^\infty z m(dz) \in [0, \infty)$, that is,

$$\mathbb{E}(e^{uJ_t}) = \exp \left\{ t \int_0^\infty (e^{uz} - 1) m(dz) \right\}, \quad t \in [0, \infty), \quad u \in (-\infty, 0].$$

We study asymptotic properties of the maximum likelihood estimator (MLE) for the growth rate b of the model based on continuous time observations $(Y_t)_{t \in [0, T]}$ as $T \rightarrow \infty$. We distinguish three cases: subcritical, critical and supercritical cases according to $b > 0$, $b = 0$ and $b < 0$. In the subcritical case we prove weak consistency and asymptotic normality, and, under the additional moment assumption $\int_0^1 z \log(\frac{1}{z}) m(dz) < \infty$, strong consistency as well. In the supercritical case, we prove strong consistency and mixed normal (but non-normal) asymptotic behavior, while in the critical case, weak consistency and non-standard asymptotic behavior are described. Concerning the asymptotic behavior of the MLE in the supercritical case, we derive a stochastic representation of the limiting mixed normal distribution, where the almost sure limit of an appropriately scaled jump-type supercritical CIR process comes into play. This is a new phenomena, compared to the critical case, where a diffusion-type critical CIR process plays a role. The full presentation of our results can be found in [1].

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On the notion of information -the 4th revolution and the info-sphere

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Information became one of the most important notion of our era. Still, there is no agreement on its definition. According to Floridi [1] we are in the 4th revolution, and a new information infrastructure, the info-sphere, is under construction for the next generation. Computation is widely accepted as information transformation. A Paradox by Denning and Bell [2] says that computers transmit and transform information without reference to its meaning. How can they generate new meaning for the observer? Where does this new information come from? Classical, formal information theory cannot answer this question. Denning's proposal to resolve the paradox is to consider information as a pair of the signs and the referents. In my approach, I extend it with a third component, the observer. So the triplet of signs (in my wordings formation), the referents and the observer/producer compose a real occurrence/instance of information. A formation may exist independently of the observer. The meaning for the observer is based on observation of former information instances and moreover, formation that carries the meaning must be sensed in parallel with the actual information instance. Information instance is a process developing in space and time like a computation. But computation produces only new formations from formations. Observing the result of a computation may lead to obtain new information. The ever evolving set of existing formations - including formations in living human brains - composes the info-sphere. The origin of formal information theories is closely related to randomness and uncertainty. In the presentation my goal is to give a new light on the role and importance of probabilistic and algorithmic information theory. Conclusion: information theories, theories of information and informatics all belong to the science of the info-sphere.

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Transfer theorems concerning asymptotic expansions for the distribution functions of statistics constructed from samples with random sizes

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In the communication, we discuss the transformation of the asymptotic expansion for the distribution of a statistic admitting Edgeworth expansion if the sample size is replaced by a random variable. We demonstrate that all those statistics that are regarded as asymptotically normal in the classical sense, become asymptotically Laplace or Student if the sample size is random. We especially separate the case where the Student distribution parameter ("the number degrees of freedom") is small. We show that the Student distribution with arbitrary "number of degrees of freedom" can be obtained as the limit when the sample size is random. We emphasize the possibility of using a family of Student distributions as a comfortable model with heavy tails since in this case many relations, in particular, a likelihood function, have the explicit form (unlike stable laws). Thus, the Laplace and Student distributions may be used as an asymptotic approximation in descriptive statistics being a convenient heavy-tailed alternative to stable laws.

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Asymptotic inference for linear stochastic differential equations with time delay

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Assume that we observe a stochastic process $(X(t))_{t \in [-r, T]}$, which satisfies the linear stochastic delay differential equation

$$dX(t) = \vartheta \int_{[-r, 0]} X(t+u) a(du) dt + dW(t), \quad t \geq 0,$$

where a is a finite signed measure on $[-r, 0]$. The local asymptotic properties of the likelihood function are studied.

In the first part of the talk, we study a special case, when the delay is uniform. Namely, when a is the Lebesgue-measure and $r = 1$. In this special model local asymptotic normality is proved in case of $\vartheta \in (-\frac{\pi^2}{2}, 0)$, local asymptotic mixed normality is shown if $\vartheta \in (0, \infty)$, periodic local asymptotic mixed normality is valid if $\vartheta \in (-\infty, -\frac{\pi^2}{2})$, and only local asymptotic quadraticity holds at the points $-\frac{\pi^2}{2}$ and 0 .

In the general model we have not chance to determinate the exact values of the parameter, where the appropriate property is valid. However we can add a sufficient condition for this. Local asymptotic normality is proved in case of $v_\vartheta^* < 0$, local asymptotic quadraticity is shown if $v_\vartheta^* = 0$, and, under some additional conditions, local asymptotic mixed normality or periodic local asymptotic mixed normality is valid if $v_\vartheta^* > 0$, where v_ϑ^* is an appropriately defined quantity. As an application, the asymptotic behaviour of the maximum likelihood estimator $\hat{\vartheta}_T$ of ϑ based on $(X(t))_{t \in [-r, T]}$ can be derived as $T \rightarrow \infty$.

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Investigation of finite-source retrial queueing systems with collisions and non-reliable server using MOSEL

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In this article, we investigate a single-server retrial queueing system, with collision [1], [2] of the customers where the server is unreliable [3]. All the random variables involved in the model are assumed to be independent and exponentially distributed. If the server is broken, we will treat two different cases. In the first, the request automatically enters into the orbit, from which it tries again after exponential intervals. In the second case, the call stays at the server and its service continues after the server has been repaired. Our interest is to give the main steady-state performance measures of the system computed by the help of the MOSEL tool. Numerous diagrams illustrate the effect of the input parameters on the mean response time.

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On tail behavior of second-order Galton–Watson processes with immigration

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We suppose that the individuals reproduce at age 1 and also at age 2, and then they die immediately. For each $n \in N$, the population consists of the offsprings born at time n and the immigrants arriving at time n . For each $n, i, j \in N$, the number of offsprings produced at time n by the i^{th} individual of the $(n-1)^{\text{th}}$ generation and by the j^{th} individual of the $(n-2)^{\text{nd}}$ generation will be denoted by $\xi_{n,i}$ and $\eta_{n,j}$, respectively, and ε_n denotes the number of immigrants in the n^{th} generation. Then, for the population size X_n of the n^{th} generation, we have

$$X_n = \sum_{i=1}^{X_{n-1}} \xi_{n,i} + \sum_{j=1}^{X_{n-2}} \eta_{n,j} + \varepsilon_n, \quad n \in N,$$

where X_{-1} and X_0 are nonnegative integer-valued random variables (the initial population sizes). Here $\{X_{-1}, X_0, \xi_{n,i}, \eta_{n,j}, \varepsilon_n : n, i, j \in N\}$ are supposed to be independent non-negative integer-valued random variables, and $\{\xi_{n,i} : n, i \in N\}$, $\{\eta_{n,j} : n, j \in N\}$ and $\{\varepsilon_n : n \in N\}$ are supposed to consist of identically distributed random variables, respectively. The stochastic process $(X_n)_{n \geq -1}$ is called a second-order Galton–Watson process with immigration. If $\varepsilon_n = 0$, $n \in N$, then we say that $(X_n)_{n \geq -1}$ is a second order Galton–Watson branching process without immigration.

The tail behavior of second-order Galton–Watson branching processes without or with immigration will be described using the methods of [1] and [2]. In the absence of immigration, we consider two cases: (i) X_0 has a regularly varying tail and $X_{-1} = 0$, and (ii) $\xi_{1,1}$ and $\eta_{1,1}$ have regularly varying tails and $(X_0, X_{-1}) = (1, 0)$. In case of immigration, we suppose that ε_1 has a regularly varying tail and the distribution of (X_0, X_{-1}) is the stationary distribution of the Markov chain (X_n, X_{n-1}) , $n \in N$.

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Testing for white noise in functional time series

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We propose a new procedure for white noise testing of a functional time series. Our approach is based on an explicit representation of the L^2 -distance between the spectral density operator and its best (L^2) -approximation by a spectral density operator corresponding to a white noise process. The estimation of this distance can be accomplished by sums of periodogram kernels and it is shown that an appropriately standardized version of the estimator is asymptotically normally distributed under the null hypothesis (of functional white noise) and under the alternative. As a consequence we obtain a simple test (using the quantiles of the normal distribution) for the hypothesis of a white noise functional process. In particular the test does neither require the estimation of a long run variance (including a fourth order cumulant) nor resampling procedures to calculate critical values. Moreover, in contrast to all other methods proposed in the literature, our approach also allows to test for “relevant” deviations from white noise and to construct confidence intervals for a measure which measures the discrepancy of the underlying process from a functional white noise process. See [1] for more details.

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Distance between a U-statistic and a normal random variable

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A probability metric is a functional that measures the distance between two random variables or distributions. Probability metrics can be used to investigate the asymptotic behavior of a statistical function or estimator. The metric approach to the problems of accuracy of approximations of distributions appeared in the theory of probability in the mid-1930s. U-statistics are generalized averages and include among others the sample mean and the unbiased sample variance as special cases. In our presentation we investigate the distance between a U-statistic based on a collection of independent identically distributed random variables and a normal random variable using Zolotarev's ideal metric [1]. The distance between a U-statistic based on a collection of dependent random variables and a normal random variable is also considered.

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On the probability of the event: at least one cell contains given number of particles

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Let $\xi_1, \xi_2, \dots, \xi_N$ be independent identically distributed integer valued non-negative random variables. The random variables $\eta_1, \eta_2, \dots, \eta_N$ fit to the generalized allocation scheme of n particles to N cells if their joint distribution can be given by the formula

$$P\{\eta_1 = k_1, \dots, \eta_N = k_N\} = P\left\{\xi_1 = k_1, \dots, \xi_N = k_N, \left| \sum_{i=1}^N \xi_i = n \right.\right\},$$

where $k_1, k_2, \dots, k_N$ are non-negative integer numbers such that $k_1 + k_2 + \dots + k_N = n$.

The generalized allocation scheme was introduced by V. F. Kolchin in [1]. Several models of discrete probability theory (urn schemes, random forests, random permutations, random allocations) can be described by generalized allocation schemes.

Let $0 < K \leq N$ be an integer. We will denote by B_r the event that in a generalized allocation scheme at least one of the first K cells contains r particles. That is $B_r = \cup_{i=1}^K A_i$, where $A_i = \{\eta_i = r\}$.

The aim of the talk is to give conditions which provide

$$P(B_r) \rightarrow 1 - e^{-\beta}, \quad \text{where } 0 < \beta < \infty.$$

We will assume that the random variables $\xi_i = \xi_i(\alpha)$, $1 \leq i \leq N$, have the following distribution

$$P\{\xi_i = k\} = \frac{b_k \alpha^k}{k! B(\alpha)}, \quad k = 0, 1, 2, \dots,$$

where $b_k \geq 0$ for all k , and the series $B(\alpha) = \sum_{k=0}^{\infty} \frac{b_k}{k!} \alpha^k$ has positive radius of convergence.

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Estimating functions for SDE driven by stable Lévy processes

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Joint work with Arnaud Gloter.

This talk is concerned with parametric inference for a stochastic differential equation driven by a pure-jump Lévy process, based on high frequency observations on a fixed time period. Assuming that the Lévy measure of the driving process behaves like that of an α -stable process around zero, we propose an estimating functions based method which leads to asymptotically efficient estimators for any value of $\alpha \in (0, 2)$ and does not require any integrability assumptions on the process. The main limit theorems are derived thanks to a control in total variation distance between the law of the normalized process, in small time, and the α -stable distribution. This method is an alternative to the non gaussian quasi-likelihood estimation method proposed by Masuda [1] where the Blumenthal-Gettoor index α is restricted to belong to the interval $[1, 2)$.

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Tail measure of regularly varying time series

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Homogeneous measures appear naturally in the theory of regular variations and sum- or max-stable distributions. We consider the stochastic representation of homogeneous measures and characterize the existence and equivalence of stochastic representation. Existence is closely related to the decomposition of the homogeneous measure into a radial and an angular component.

In the time series framework, we establish connections with the so-called time-change formula by Basrak and Segers [1] and non-singular flow representations of stable process by Rosinski [2]. We also provide a construction, closely related to the de Haan representation of max-stable process, for a stationary regularly varying time series with a given tail measure. Properties of the constructed process are investigated, such as mixing, anticlustering and existence of a tail index.

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VAR model based clustering method for multivariate time series data

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Classification of time series data is very important in many fields of scientific research, popular examples being medical science, climate studies and geographical scenarios. Notable studies in this regard can be found in [1], [4], [5], [6], [7], [8]. In the current study, the focus is on multivariate time series data. In practical situation, such problems can arise in finance, economics, control theory and health process. In order to address these problems, one of the most popular technique is the vector auto-regression (VAR) process. As the main interest is to develop a classification method for multivariate time series data, we will use the VAR models in our algorithm.

Suppose, $\{\mathbf{X}_i, i = 1, \dots, k\}$, is the set of k time series data, each object being a $n \times p$ matrix generated from a VAR process. Recall that the vector of the least squares estimates of all the parameters in a VAR model is consistent and asymptotically normally distributed. The main contribution of this paper is three-fold. First, borrowing the idea of Degras et al. [3], we propose to use a simulation based approach to develop a method to test if two multivariate time series are coming from same VAR process. Second, the testing method is extended to a group of time series objects. And finally, a new clustering algorithm is developed using the testing methods. The proposed algorithm does not use a predetermined number of clusters and finds the best possible clustering from the data.

One of the main advantages of using the simulation based method is that it provides better finite sample properties. We have showed that, for time series with length 50, the empirical power of the testing method and the classification algorithm are more than 0.8, and the power reaches nearly 1 for lengths greater than 100. We have further proved that the clustering algorithm we devised is consistent, in the sense that the algorithm reaches the ground truth when the number of observations in each time series approaches infinity. As a practical example, the method is implemented to identify activities of different persons from a real life data obtained from single chest-mounted accelerometer. (cf. [2]).

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Statistical analysis of TripAdvisor ratings using entropy and mixture distributions

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In this work, a random sample of 1200 rating datasets from TripAdvisor website analyzed using the discrete entropy measure. The sample include ratings of Hotels, Restaurants and Thing to Do in Berlin, Munich, London, Paris, Prague. The Binomial distribution suggested as the basis of data empirical distribution. Calculating the entropy of each data set the theoretical approach of Binomial distribution extended to mixture of two Binomials (double Binomial) and to Combined Uniform and Binomial (CUB model). The calculation of ratings datasets entropy used to explain the necessity of transition from simple Binomial to mixture distributions. The used method of entropy calculations illustrates the disorder of ratings (disagreement of raters) and explain the error of this approximation measured by Dissimilarity Index or more appropriate called for this type of data False Rate criterion. The results of entropic analysis of online rating for the 5 European Cities used in this study, discussed further and an assessment of those cities as tourism destinations produced.

Keywords: Entropy, Binomial Mixture, CUB model, Online Rating, TripAdvisor, Europe tourism destinations.

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Investigation of the window variance noise component of the multicomponent signals

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Signal segmentation is an essential part of signal processing and may serve as a tool for advanced signal interpretation and data classification. Our research is focused on the investigation of the window variance noise component as a random variable for a multicomponent signal model.

For each measuring point τ_k , let's consider the value of signal C as a sum of physically independent true values of underlying processes $A_l, l = 1, 2, \dots, m$, and statistically independent noise terms $\xi_1, \xi_2, \dots, \xi_m$ for each true signal component, respectively. The random variable $\xi_{l,k}$ is supposed to be normally distributed having zero mean and variance $\sigma_{l,k}^2, l = 1, 2, \dots, m$, and for each τ_k for the fixed true processes $A_l, l = 1, 2, \dots, m$ random variables $\xi_{k,l}$ are independent identically distributed variables.

Theorem. For measuring signal $C(t)$ for each $\tau_k, k = \{1, 2, \dots, r\}$ window variance noise component W_n^Ξ is represented by:

- (i) random variable $\sum_{l=1}^m W_{l,n}^\xi \sim \Gamma\left(\frac{n}{2\sigma_l^2}, \frac{n-1}{2}\right), \text{ if } (A_{l,j} - \bar{A}_{l,n}) = 0$
 $\forall l \in \{1, 2, \dots, m\}, \forall j \in \{0, 1, \dots, n-1\},$
- (ii) sum of random variables $\sum_{l=1}^m \sum_{j=0}^{n-1} \frac{1}{n} (\xi_{l,j}^2 + \xi_{l,j} (2A_{l,j} - 2\bar{A}_{l,n})) - \sum_{l=1}^m \bar{\xi}_{l,n}^2,$
 $\text{if } \exists l \in \{1, 2, \dots, m\} \text{ and } j \in \{0, 1, \dots, n-1\} : (A_{l,j} - \bar{A}_{l,n}) \neq 0, \text{ and } \bar{\xi}_{l,n}^2 \sim \Gamma\left(\frac{n}{2\sigma_l^2}, \frac{1}{2}\right),$
 and characteristic function of $\sum_{l=1}^m \frac{1}{n} \sum_{j=0}^{n-1} (\xi_{l,j}^2 + \xi_{l,j} (2A_{l,j} - 2\bar{A}_{l,n}))$ is expressed as

$$\varphi(t) = \prod_{l=1}^m \left(1 - \frac{2it\sigma_l^2}{n}\right)^{-\frac{n}{2}} \prod_{j=0}^{n-1} e^{\frac{(A_{l,j} - \bar{A}_{l,n})^2 t^2 \sigma_l^2}{itn\sigma_l^2 - \frac{n^2}{2}}}.$$

Our theoretical findings are consistent with the previously obtained empirical characteristics of the window variance noise component and are expected to be of potential use for the signal segmentation and predicting.

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General Bahr-Esseen inequalities and their applications

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Let $1 \leq p \leq 2$ and let X_n , $n = 1, 2, \dots$ be a sequence of independent random variables (r.v.'s) with finite p -th moment and mean zero. Then

$$E \left| \sum_{k=1}^n X_k \right|^p \leq c_{p,n} \sum_{k=1}^n E |X_k|^p \quad (1)$$

for all $n = 1, 2, \dots$, where $c_{p,n} \leq 2 - n^{-1}$. Inequality (1) is the p -th von Bahr-Esseen moment inequality, [3]. We remark that (1) is obviously true for $0 < p \leq 1$.

We shall show that the Bahr-Esseen inequality holds with exponent p , if it holds with exponent q for the truncated and centered random variables if $q > p$. We mention that Theorem 2.1 in [2] is a Bahr-Esseen's inequality for pairwise independent r.v.'s. We shall apply the method presented in [2].

Let $\eta_1, \eta_2, \dots, \eta_n$ be a sequence of r.v.'s. Consider the condition

$$E e^{\sum_{i=1}^n \lambda \eta_i} \leq g(n) \prod_{i=1}^n E e^{\lambda \eta_i}. \quad (2)$$

If condition (2) is satisfied for $g(n) = 1$ and for all $\lambda \in R$, then $\eta_1, \eta_2, \dots, \eta_n$ are called acceptable, see [1].

We shall prove that if relation (2) is satisfied for the truncated random variables, then an exponential inequality is true, which implies Rosenthal's inequality and also von Bahr-Esseen's moment inequality.

Then we apply the results to obtain weak and strong laws of large numbers and complete convergence.

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Density in small time for Lévy driven SDE and application to the LAMN property

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This work focuses on the Local Asymptotic Mixed Normality (LAMN) property for high frequency observations of a continuous time process solution of a stochastic differential equation driven by a pure jump Lévy process L . It is assumed that the Lévy measure of L is similar to the one of an α -stable process in a neighbourhood of zero. The process is observed on the fixed time interval $[0,1]$ and the parameters appear in both the drift coefficient and scale coefficient. We compute the asymptotic Fisher information of the model and find that the rate in the LAMN property depends on the index α . This extends the results of [1] where α is restricted to be in $(1,2)$ and the parameter appears only in the drift coefficient. The proof relies on the small time asymptotic behaviour of the transition density of Lévy driven SDE obtained in [2].

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Convergence results for Oppenheim Series expansions and Oppenheim Continued Fractions expansions

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Let $(B_n)_{n \geq 1}$ be a sequence of integer valued random variables defined on $(\Omega, \mathcal{A}, P)$, where $\Omega = [0, 1]$, $\mathcal{A}$ is the σ -algebra of the Borel subsets of $[0, 1]$ and P is the Lebesgue measure on $[0, 1]$. We are also given a density f on $(0, 1)$, a sequence of functions $\varphi_n : \mathbb{N}^* \rightarrow \mathbb{R}^+$ and a sequence of nonnegative numbers $(y_n)_{n \geq 1}$ with $y_n = y_n(h_1, \dots, h_n)$ (i.e. possibly depending on the n integers $h_1, \dots, h_n$) such that, for $h_1 \geq 1$ and $h_j \geq \varphi(h_{j-1})$, $j = 2, \dots, n$

$$P(B_{n+1} = h_{n+1} | B_n = h_n, \dots, B_1 = h_1) = \int_{\alpha_n}^{\beta_n} f(u) du, \quad (1)$$

where we set

$$\delta_j(h, k, y) = \frac{\varphi_j(h)(1+y)}{k + \varphi_j(h)y}, \quad j = 1, \dots, n$$

$$\alpha_n = \delta_n(h_n, h_{n+1} + 1, y_n), \quad \beta_n = \delta_n(h_n, h_{n+1}, y_n).$$

The above relation (1) doesn't necessarily say that $(B_n)_{n \geq 1}$ is a Markov chain, since the y_n may depend on (some of) the integers $h_1, \dots, h_{n-1}$.

Let $Y_n = y_n(B_1, \dots, B_n)$ and

$$R_n = \frac{B_{n+1} + \varphi_n(B_n)Y_n}{\varphi_n(B_n)(1 + Y_n)}, \quad S_n = \sum_{k=1}^n R_k.$$

Our main result is the following

Theorem. Assume that each φ_n takes values in $[1, +\infty)$ and let $f : [0, 1] \rightarrow \mathbb{R}^+$ be bounded and such that

$$\lim_{x \rightarrow 0} f(x) =: \alpha \in \mathbb{R}^+.$$

Then

$$\frac{S_n}{n \log n} \xrightarrow{P} \alpha, \quad n \rightarrow \infty.$$

We apply this result to Oppenheim Series expansions and to Oppenheim Continued Fractions expansions.

Probabilistic temperature forecasting with a heteroscedastic model

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Today, weather prediction is based on some numerical weather prediction (NWP) model which is run multiple times, each time with variations in the model parameterizations and/or initial and boundary conditions. To correct the resulting ensemble forecasts for biases and dispersion errors, statistical postprocessing models are typically employed, many of them being extensions of the state-of-the-art Ensemble Model Output Statistics (EMOS) model. In [1], a novel postprocessing model has been developed which utilizes the autoregressive information present in the forecast errors of the raw ensemble members to set up an EMOS-like predictive distribution. This AR-EMOS approach uses the autoregressive models of the individual forecast errors to create an AR-adjusted forecast ensemble, which is the basis for estimating the EMOS predictive distribution.

In this work, we propose an extension of the original AR-EMOS model. The extension incorporates a heteroscedastic variance parameter that is able to account for the well-known spread-error correlation of forecast ensembles, which means that there is a positive association between the forecast error of the ensemble mean and the spread of the ensemble. To further improve predictive performance, an additional high-resolution forecast is used as covariate in the AR-EMOS model. In a case study with 24-h temperature forecasts of the European Center for Medium-Range Weather Forecasts (ECMWF) over Germany the extended heteroscedastic AR-EMOS shows improved predictive performance in comparison to the standard EMOS approach.

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Transition densities of Lévy processes with slowly varying symbols

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One of the well-known results of the asymptotic behaviour of the transition density of pure-jump Lévy processes originates from the twenties of the last century (Pólya (1923), Blumenthal, Gettoor (1961)) and it concerns the transition density $p_t^{(\alpha)}(x)$ of the symmetric α -stable process X_α in $\mathbf{R}^d$. In this talk we present recently obtained results for transition densities of isotropic unimodal Lévy processes. Let $\mathbf{X} = (X_t : t \geq 0)$ be an isotropic unimodal Lévy process on $\mathbf{R}^d$ with the characteristic exponent ψ . In the paper by Bogdan, Grzywny and Ryznar (2014), the estimates of transition densities $p(t, x)$ were studied under the assumption that ψ has lower and upper Matuszewska indices strictly between 0 and 2. More detailed information can be obtained whenever stronger assumption about the behaviour of ψ is imposed. Namely, in the paper by Cygan, Grzywny and Trojan (2017), it is proved an asymptotic behaviour when ψ varies regularly at infinity with index $\alpha \in (0, 2)$.

The natural question arises about the asymptotic behaviour and estimates of the semigroup for the endpoints $\alpha \in \{0, 2\}$. In this talk we shed some light on the situation $\alpha = 0$. We provide a solution for a large class of slowly varying symbols including geometric stable and iterated geometric stable cases. Namely, we study processes with symbols belonging to de Haan class Π_ℓ^∞ associated to a function ℓ slowly varying at infinity which is equivalent to regular variation with index $-d$ at the origin of the density of Lévy's measure of the process. We derive several asymptotic results describing the behaviour of transition densities $p(t, x)$. Moreover, we present several estimates for $p(t, x)$ under various assumptions for isotropic unimodal processes, which improve the existing results even in the case of subordinate Brownian motion (see [1]).

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Some results for demimartingales and for randomly indexed demimartingales

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We consider a sequence of strong demimartingales. For these random objects, a central limit theorem is obtained by utilizing Zolotarev's ideal metric and the fact that a sequence of strong demimartingales is ordered with the sequence of its independent duplicates via the convex order. The central limit theorem obtained is also valid for demimartingales and this result is considered to be the first central limit theorem for demimartingales available in the literature. Furthermore, for the class of demimartingales a Whittle-type inequality is proved that generalizes a result provided by Prakasa Rao (2002). The Whittle-type inequality is instrumental for obtaining several maximal and moment inequalities. Finally, the family of randomly indexed demimartingales is introduced and for this class of random variables some asymptotic results are presented.

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A comparison of high-dimensional sample covariance and correlation matrices of a heavy-tailed time series

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In Principal Component Analysis one studies the sample covariance or sample correlation matrix, both of which often lead to the same result. In this talk, we first analyze the joint distributional convergence of the largest eigenvalues of the sample covariance matrix of a p -dimensional heavy-tailed time series when p converges to infinity together with the sample size n . Assuming a regular variation condition with tail index $\alpha < 4$, we employ a large deviations approach to show that the extreme eigenvalues are essentially determined by the extreme order statistics from an array of iid random variables. The asymptotic behavior of the extreme eigenvalues is then derived routinely from classical extreme value theory. The resulting approximations are strikingly simple considering the high dimension of the problem at hand.

Then we compare the behavior of the eigenvalues of the sample covariance and sample correlation matrices and argue that the latter seems more robust, in particular in the case of infinite fourth moment.

We show that the largest and smallest eigenvalues of a sample correlation matrix stemming from n independent observations of a p -dimensional time series with iid components converge almost surely to $(1+\sqrt{\gamma})^2$ and $(1-\sqrt{\gamma})^2$, respectively, as $n \rightarrow \infty$, if $p/n \rightarrow \gamma \in (0, 1]$ and the truncated variance of the entry distribution is “almost slowly varying”, a condition we describe via moment properties of self-normalized sums. Moreover, the empirical spectral distributions of these sample correlation matrices converge weakly, with probability 1, to the Marčenko–Pastur law.

Key Words: PCA, regular variation, sample covariance matrix, sample correlation matrix, extreme eigenvalues, spectral distribution.

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On refinement of one limit theorem in theory of critical branching processes

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Let $\{Z_n, n \geq 0\}$ denote the Galton-Watson branching process having offspring probability generating function $F(s) = \sum_{j \geq 0} p_j s^j$. Assume that $p_0 > 0$ and the mean per-capita offspring number is $\sum_{j \geq 1} j p_j = 1$, i.e. the process is critical. We also suppose that

$$F(s) = s + (1-s)^{1+\nu} \mathcal{L}\left(\frac{1}{1-s}\right) \quad [\mathfrak{R}_\nu]$$

for $0 \leq s < 1$, where $0 < \nu < 1$ and the function $\mathcal{L}(\cdot)$ is slowly varying at infinity. Evidently, $\mathbb{E}Z_n^2 = \infty$ provided that $[\mathfrak{R}_\nu]$ holds. Let the variable $\mathcal{H}$ be an extinction time of the process and $Q_n := \mathbf{P}\{\mathcal{H} > n\}$. Slack [2], [3] has proved that distribution function $G_n(x) := \mathbf{P}\{Q_n Z_n \leq x | \mathcal{H} > n\}$ converges weakly to a nonnegative limit law which has the Laplace transform in the form of $\Psi(\theta) = 1 - (1 + \theta^{-\nu})^{-1/\nu}$.

In this report we find out a rate of convergence in the Slack's theorem provided that the function $\mathcal{L}(\cdot)$ is slowly varying with remainder, i.e.

$$\frac{\mathcal{L}(\lambda x)}{\mathcal{L}(x)} = 1 + o\left(\frac{\mathcal{L}(x)}{x^\nu}\right) \quad \text{as } x \rightarrow \infty; \quad [\mathcal{L}_\nu]$$

see [1, p.185].

Theorem. *If the conditions $[\mathfrak{R}_\nu]$ and $[\mathcal{L}_\nu]$ hold, then*

$$\sup_{\theta > 0} \left| \int_{x>0} e^{-\theta x} dG_n(x) - \Psi(\theta) \right| = \frac{1+\nu}{2\nu^2} \frac{\ln n}{n} (1 + o(1))$$

as $n \rightarrow \infty$.

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Limit theorems for critical branching processes with immigration in VE

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Let $\{\xi_{k,j}, \varepsilon_k : k, j \in \mathbb{N}\}$ be independent, non-negative, integer-valued random variables such that $\{\xi_{k,j} : j \in \mathbb{N}\}$ are identically distributed for each $k \in \mathbb{N}$. Define

$$X_k = \sum_{j=1}^{X_{k-1}} \xi_{k,j} + \varepsilon_k \quad \text{for } k \in \mathbb{N}, \quad X_0 = 0.$$

Assume that, for all $k \in \mathbb{N}$, $m_k := \mathbb{E}(\xi_{k,1})$, $\lambda_k := \mathbb{E}(\varepsilon_k)$, $\sigma_k^2 := \text{Var}(\xi_{k,1})$, $b_k^2 := \text{Var}(\varepsilon_k)$ are finite. The sequence $(X_k)_{k \in \mathbb{Z}_+}$ is called a branching process with immigration (BPI) in varying environment (VE). A BPIVE is called nearly critical if $m_k = 1 + \alpha k^{-1} + \delta_k$ for some $\alpha \in \mathbb{R}$, where $\sum_{k=1}^{\infty} |\delta_k| < \infty$. The parameter α is called the index of criticality. In [1] functional limit theorems are proved for the strongly critical case, i.e., when $\alpha = 0$.

In the talk the general case is considered. It is proved that if $\lambda := \lim_{k \rightarrow \infty} \lambda_k$ then the following asymptotic behavior holds for the expectation of the process: (i) $n^{-1}E(X_n) \rightarrow \lambda(1 - \alpha)^{-1}$ if $\alpha < 1$; (ii) $(n \ln n)^{-1}E(X_n) \rightarrow \lambda$ if $\alpha = 1$; (iii) $n^{-\alpha}E(X_n) \rightarrow \mu$ if $\alpha > 1$, where $\mu \in \mathbb{R}_+$ is a constant, as $n \rightarrow \infty$. Functional limit theorem and fluctuation limit theorem are also investigated for the random step process $\mathcal{X}_t^n := X_{\lfloor nt \rfloor}$. Under the assumptions $\alpha < 1$ and $\sigma_k^2 \rightarrow \sigma^2$ we have $n^{-1}\mathcal{X}^n \xrightarrow{\mathcal{L}} \mathcal{X}$ as $n \rightarrow \infty$, where $(\mathcal{X}_t)_{t \in \mathbb{R}_+}$ satisfies the inhomogeneous SDE

$$d\mathcal{X}_t = (\lambda + \alpha t^{-1}\mathcal{X}_t) dt + \sigma \sqrt{\mathcal{X}_t} d\mathcal{W}_t, \quad t > 0,$$

where $(\mathcal{W}_t)_{t \in \mathbb{R}_+}$ is a standard Wiener process, with initial condition $\mathcal{X}_0 = 0$. Moreover, $n^{-1/2}(\mathcal{X}^n - \mathbb{E}(\mathcal{X}^n)) \xrightarrow{\mathcal{L}} \tilde{\mathcal{X}}$ as $n \rightarrow \infty$ under the assumptions $\alpha < 1/2$ and $k\sigma_k^2 \rightarrow \sigma^2$ where $(\tilde{\mathcal{X}}_t)_{t \in \mathbb{R}_+}$ is an inhomogeneous Ornstein-Uhlenbeck type process.

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Asymptotic properties of Kendall random walks

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We consider discrete time Lévy processes in the generalized convolution sense (see [2]), which are extremal Markovian sequences of the Kendall type introduced in [3], i.e. stochastic process defined by:

$$X_0 = 1, \quad X_1 = Y_1, \quad X_{n+1} = M_{n+1} [\mathbf{1}(\xi_n > \varrho_{n+1}) + \theta_{n+1} \mathbf{1}(\xi_n < \varrho_{n+1})],$$

where

$$M_{n+1} = \max \{X_n, Y_{n+1}\}, \quad \varrho_{n+1} = \frac{\min \{X_n, Y_{n+1}\}^\alpha}{\max \{X_n, Y_{n+1}\}^\alpha},$$

and sequences $(Y_k) \sim i.i.d.(\nu)$, $(\xi_k) \sim i.i.d.(U([0, 1]))$, $(\theta_k) \sim i.i.d.(\pi_{2\alpha})$ are independent.

Recall that the Kendall convolution algebra is the world, where:

$$\delta_x \triangle_\alpha \delta_1 = x^\alpha \pi_{2\alpha} + (1 - x^\alpha) \delta_1, \quad x \in [0, 1], \quad \pi_{2\alpha}(dx) = 2\alpha x^{-2\alpha-1} \mathbf{1}_{[1, \infty)}(x) dx.$$

We prove Central Limit Theorem in the Kendall convolution algebra using the Williamson transform (see [1],[4]). We notice that obtained stable distributions belong to maximal domain of attraction of the Fréchet distribution. We also prove convergence of finite dimensional distributions of the following process:

$$Z_n(t) = n^{-1/\alpha} X_{[nt]}.$$

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Optimal change point tests in high dimension

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Suppose we observe a high-dimensional time series and are interested in testing for change-points. We provide tests that are minimax optimal subject to sparse alternatives. The construction is based on consistent bootstrap methods or appropriate limit distributions. The conditions are very flexible and include many popular multivariate linear and nonlinear models from the literature, such as ARMA, GARCH and related models. In the case of general factor models our tests are even adaptive with respect to the spatial dependence structure, and the corresponding minimax rate can be expressed in terms of underlying quantiles.

On asymptotic behavior of critical branching processes with dependent immigration

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Consider two independent sequences $\{\xi_{k,j}, k, j \in \mathbf{N}\}$ and $\{\tau_k, k \in \mathbf{N}\}$ of non-negative, integer-valued random variables. Assume that $\xi_{k,j}, k, j \in \mathbf{N}$ are independent and identically distributed and $\{\tau_k, k \in \mathbf{N}\}$ is the sequence of m -dependent random variables.

We consider of branching process with immigration (see [1]) $Z_k, k = 0, 1, \dots$ defined by the recursion

$$Z_k = \sum_{j=1}^{Z_{k-1}} \xi_{k,j} + \tau_k, \quad k \in \mathbf{N}, Z_0 = 0.$$

We are interested in the asymptotic behavior of Z_n as $n \rightarrow \infty$ in the critical case i.e. $E\xi_{k,j} = 1$. We proved the following theorem.

Theorem. Suppose that the following conditions are satisfied:

1) $E\xi_{1,1} = 1, \text{Var}\xi_{1,1} < \infty$;

2) $\lambda_k = E\tau_k = k^\alpha L_\alpha(k), b_k^2 = \text{Var}\tau_k = k^\beta L_\beta(k), k \in \mathbf{N}$, where $\alpha, \beta \geq 0$ fixed numbers, $L_\alpha(n)$ and $L_\beta(n)$ are slowly varying functions and $\lambda_n \rightarrow \infty, b_k^2 = o(n\lambda_n^2)$ as $n \rightarrow \infty$.

Then, for any $T > 0$

$$\sup_{0 \leq t \leq T} \left| \frac{Z_{[nt]}}{n^{1+\alpha} L_\alpha(n)} - \frac{t^{1+\alpha}}{1+\alpha} \right| \xrightarrow{P} 0 \text{ as } n \rightarrow \infty.$$

Here $[a]$ denotes the integer part of number a , and symbol $\xrightarrow{P}$ denotes the convergence in probability.

When the immigration process $\{\tau_k, k \in \mathbf{N}\}$ is a sequence of independent random variables, asymptotic the processes $Z_{[nt]}$ was studied in [2].

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Maximum likelihood estimation for Wishart processes

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(joint work with Aurélien Alfonsi and Clément Rey, École des Ponts-CERMICS, France)

In the last decade, there has been a growing interest to use Wishart processes for modelling, especially for financial applications. However, there are still few studies on the estimation of its parameters. Here, we study the Maximum Likelihood Estimator (MLE) in order to estimate the drift parameters of a Wishart process. We obtain precise convergence rates and limits for this estimator in the ergodic case and in some nonergodic cases. We check that the MLE achieves the optimal convergence rate in each case. Motivated by this study, we also present new results on the Laplace transform that extend the recent findings of Gnoatto and Grasselli and are of independent interest.

Hausdorff dimension results for self-affine random fields

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We present a close connection between the Hausdorff dimension of the graph for general multidimensional self-affine random fields and the carrying dimension of the corresponding random occupation measure. The latter has been investigated by U. Zähle in a series of papers around 1990. In case self-affinity of the random field can be expressed by exponential scaling operators, these dimensions can be explicitly calculated by means of the singular value function under a mild regularity assumption. The results partially extend to the Hausdorff dimension of the range of self-affine random fields.

On the tail of the solution of random fixed point equations

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Consider the perpetuity equation $X \stackrel{\mathcal{D}}{=} AX + B$, where (A, B) and X on the right-hand side are independent. The Kesten–Grincevičius–Goldie theorem states that $P\{X > x\} \sim cx^{-\kappa}$ if $\log A$ is nonarithmetic, $EA^\kappa = 1$, $EA^\kappa \log_+ A < \infty$, and $E|B|^\kappa < \infty$. We assume that $E|B|^\nu < \infty$ for some $\nu > \kappa$, and consider two cases (i) $EA^\kappa = 1$, $EA^\kappa \log_+ A = \infty$; (ii) $EA^\kappa < 1$, $EA^t = \infty$ for all $t > \kappa$. We show that under appropriate additional assumptions $P\{X > x\} \sim cx^{-\kappa}\ell(x)$ holds, when the distribution of $\log A$ is nonarithmetic, while in the arithmetic case $P\{X > x\} \sim cx^{-\kappa}q(x)\ell(x)$, where ℓ is a nonconstant slowly varying function and q is an oscillating, logarithmically periodic function $q(xe^h) = q(x)$, $x > 0$, with h being the span of the arithmetic distribution of $\log A$. Thus the tail behavior in the two cases is very different. We use Goldie’s renewal theoretic approach, which allows us to extend these results to more general fixed point equations.

Universal approach to obtain Stein–Chen-like moment identities, characterizing and generalized moment identities of higher orders and their applications

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A collection of known characterizing moment identities (of the first and higher orders) for classical distributions in probability and statistics followed by a new universal approach to get such (known and new) identities (useful, e. g., in stability problems) for one-dimensional distributions is presented.

A series of new higher order generalized moment identities based on classical “probabilistic” orthogonal polynomials is presented as well, and their application to the linearization problem for products of such polynomials (useful, e. g., in combinatorics) is described.

The research is fulfilled in the framework of RAS program No 0014-2016-0038.

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Quality of service of telecommunication system with nonhomogeneous input traffic

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The problem of quality of service estimation is the most important one in telecommunication systems analysis. For many years these estimates were made on the basis of Poisson model or close to it Markov models. In the early nineties on the basis of measurements of traffic carried out at Bell laboratories it was discovered that traffic modern telecommunication systems has a completely new properties. Later it was confirmed in many other empirical studies. It has been discovered that such traffic has three important new properties: it is self-similar in a wide range of scales of measurements, has the property of long-term dependence and magnitude of the load, coming from separate sources, has a distribution with heavy tails. It has been shown that if you do not take into account these features, it leads to serious errors in the evaluation of service quality.

In our report using the methodology proposed in the paper of Norros [1] we propose some lower asymptotic estimate of the overflow probability of large buffer when the input is a stream consisting of two independent components: the fractional Brownian motion and stable Levy motion with the same Hurst parameters. In subsequent work we will consider the case when incoming stream consists of several independent components with different Hurst parameters.

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On multivariate nonparametric methods based on interdirections

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Many economic and other applications involving vector variables ask for nonparametric tests that could be used without restrictive distributional assumptions. Quite a few such tests already described in the literature are based on the popular Randles' concept of interdirections. We discuss some of these approaches and also ways how the definition of interdirections can be modified, and comment on the benefits and application of such modifications. These are the very first preliminary results achieved in collaboration with Miroslav Šiman and Davy Paindaveine.

The research is supported by the Czech Science Foundation project GA17-07384S.

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Influence in high-dimensional linear regression

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We consider a linear regression model with p parameters and n observations where both p and n are large with $n \gg p$. Although a single or small number of observations will typically have negligible influence on the least squares estimates, a larger group of observations may have a more profound influence than that suggested by the individual influences of each observation within the group. In this talk, we will discuss a measure of influence for a group of observations, which is an extension of the “hat” diagonals used to measure the influence of single observations. In addition, we will consider approaches for approximating this measure of influence when both p and n are large.

Properties of statistical procedures for network structures identification

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Network analysis become popular tool of complex objects analysis last decade. From practical point of view main problem of network analysis is to identify key network structures such as maximum spanning tree [1], threshold graph [2], gaussian graphical model [3, 4] by observations. In the paper properties of procedures from [1], [2], [4] such as optimality, unbiasedness, invariance for network structure identification are investigated. Problem of network structures identification is formulated in the framework of random variables network [5] as multiple decision problem [6]. It is proved that procedures for gaussian graphical model identification and selection by expectations are optimal unbiased procedures. Also it is shown that procedure for threshold graph identification is optimal in the class of invariant procedures with respect to group of shift and scale transformations. Moreover it is proved that statistical procedure for maximum spanning tree identification [1] is invariant with respect to group of shift and scale transformations.

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Discrete alternative of the Weibull distribution and a two-step grid decomposition approach

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A problem related to the Bernoulli trials with a random probability of success is considered. Within this framework a "random" analog of the classical Poisson theorem is constructed for the π -mixed binomial distributions, in which the limit distribution turns out to be mixed Poisson. Special attention is paid to the case where the mixing is performed with respect to the Weibull distribution. The corresponding mixed Poisson distribution called Poisson–Weibull law is proposed as a discrete analog of the Weibull distribution.

Some properties of the Poisson–Weibull distribution are discussed. It is shown that this class can be represented as mixed geometric distribution.

A two-stage grid algorithm is proposed for the estimation of parameters of mixed Poisson distributions, and, in particular, of Poisson–Weibull distribution. Statistical estimators for the upper bound of the grid are constructed. Examples of practical computations by the proposed algorithm are presented.

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Random matrices and orthogonal polynomials

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P. Forrester and A. Gamburd proved that under very general conditions the expectation of the characteristic polynomial of a random hermitian matrix or a sample covariance matrix coincides with known orthogonal polynomials. Interestingly the limiting law of the empirical distribution of the zeros of these polynomials is the same as that of the eigenvalues of corresponding random matrices. In my talk I wish to explore this connection and present some results, co-authored by my advisor György Michaletzky [1,2], about the moments of the zeros of these polynomials and I wish to talk about the role of the Vandermonde determinant in this theory.

Lastly I will talk about my freshly submitted work of generalization of some results of Nechita, which state that the eigenvalues of random density matrices of the form $XX^*/\text{Tr}(XX^*)$ admit the same kind of limit properties as those of XX^* after proper scaling, if the distribution of the matrix elements is Gaussian. These results were generalized for a larger class of matrices using results of Yaskov and Bai.

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Optimization of probability-of-error criterion in the models with non-Gaussian noise

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Nonparametric estimation of the signal function by the means of wavelet methods usually assumes that this function belongs to a certain regularity class. This assumption results in a conclusion about the “sparsity” of its wavelet representations, i.e. that most wavelet coefficients of the useful signal are small in their absolute value. Such considerations usually lead to the so-called thresholding procedures, when it is assumed that small coefficients contain only noise and they are set to zero. In the models with Gaussian noise such procedures are well studied and different “optimal” thresholds for the mean-square cost function are well known [1].

We consider the model of wavelet coefficients contaminated by the noise with non-necessary Gaussian distribution. For denoising we consider most popular hard and soft thresholding procedures. For the class of Lipschitz regular functions we derive the relations, which allow calculating the asymptotically optimal threshold, minimizing the probability-of-error criterion [2,3], and estimate the order of the cost function based on this criterion. We also provide some examples of calculating the optimal threshold for some specific distributions of noise coefficients.

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Numerical analysis of retrial queueing systems with conflict of customers

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In this paper we consider a closed retrial queueing system with finite number of customers. In case of an arriving request to the busy server two modes are investigated: the job is transferred to the orbit (no conflict) or the job under service is interrupted and both of them goes to the orbit (conflict). The jobs in orbit can retry reaching the server after a random time. These types of systems can be solved by numeric, asymptotic and simulation methods [1], [2]. Here a new approach of a algorithmic solution will be provided for calculating the steady-state system probabilities by the help of which the main performance measures can be computed.

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Testing parameter constancy in $I(2)$ cointegrated VAR models

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This paper explores methodology for testing parameter constancy in $I(2)$ cointegrated vector autoregressive (CVAR) models developed by Johansen (1997, 2006). A new class of likelihood-based test statistics for parameter stability is introduced in the $I(2)$ CVAR framework. This study then proves that their asymptotic distributions are non-standard but free of any nuisance parameters, so it is feasible to approximate the distributions by simulation. Selected quantiles of the approximate distributions are presented as statistical tables for applied use. Furthermore, Monte Carlo experiments are performed to examine finite-sample properties of the stability tests. These experiments provide evidence for the practicality of the proposed likelihood-based tests in empirical research on $I(2)$ data.

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Baum-Katz type theorems for random fields with dependence structure

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Complete convergence, convergence rates for non identically distributed, negatively dependent and martingale random fields are studied by application of Fuk-Nagaev inequality. The results are proved in asymmetric normalization case i.e. for the sequence equal $n_1^{\alpha_1} \cdot n_2^{\alpha_2} \cdot \dots \cdot n_d^{\alpha_d}$, where $(n_1, n_2, \dots, n_d) = \mathbf{n} \in \mathbb{N}^d$ and $\min_{1 \leq i \leq d} \alpha_i \geq \frac{1}{2}$. We discuss more general context and possibility an extension this approach to other dependent structures.

Gaussian approximation of multichannel networks with different input structure

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Most studies in queueing theory are based on the assumption that an input flow is the standart Poisson one. However, in many situations there are deviations of real flows from the standart Poisson. For example, the rate of the flow can be dependent on time, but under certain natural conditions the flow remains close to a nonhomogeneous Poisson process. Such a situation occurs, in particular, when customers are arriving on the input from various sources, and the total flow is the sum of flows with low rate.

In the work multi-channel queueing networks with different structure of an input are considered. At first, we consider an open queueing network with a nonhomogeneous Poisson flow of customers arriving from outside. Each node operates as a multi-channel queueing system. Once the service is completed at a node, the customer is transferred to another node or it leaves the network with correspondent probabilities. It is proved that under some heavy traffic assumptions on network parameters the multi-channel service process that is the number of customers at the nodes converges to a Gaussian process in the uniform topology. Correlation characteristics of the Gaussian process are written via network parameters in an explicit form.

Secondly, we suppose the input flow in the network is a sum of low rate flows from many sources. For this case, a functional limit theorem on approximation of the service process by a Gaussian process is proved, provided that the appropriate convergence conditions of the total flow to the nonhomogeneous Poisson flow are satisfied.

Finally, the previous model is replaced by a closed finite-source network. Each separate source generates a request and sends it to the input of the network. The source that sent the request for service is in standby mode and does not generate new one until its request service is completed. For such a network a similar result is formulated and proved on condition that the number of sources increases asymptotically.

Renewal theory for Kendall random walks

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We deal with the renewal theory for a class of extremal Markov chains with respect to a generalized convolution. Borowiecka-Olszewska, Jasiulis-Góldyn, Misiewicz and Rosiński (2015) [Lévy processes and stochastic integral in the sense of generalized convolution. *Bernoulli* **24**(1), 2513–2551] defined Markov processes generated by generalized convolutions. We consider generalized convolution called Kendall convolution defined by:

$$\delta_x \Delta_\alpha \delta_1 = x^\alpha \pi_{2\alpha} + (1 - x^\alpha) \delta_1, \quad x \in [0, 1],$$

where $\pi_{2\alpha}(dx) = 2\alpha x^{-2\alpha-1} \mathbf{1}_{[1, \infty)}(x) dx$.

Jasiulis-Góldyn, Misiewicz (2015) [Classical definitions of the Poisson process do not coincide in the case of weak generalized convolutions. *Lith. Math. J.* **55**(4), 518–542] introduced Poisson processes in the generalized convolution algebras. Here we consider renewal processes for any unit step distributions.

For Kendall random walk $\{X_n : n \in \mathbb{N}_0\}$ we define Δ_α - renewal process $\{N(t) : t \geq 0\}$ as follows:

$$N(t) = \begin{cases} \inf\{n : X_{n+1} > t\} \\ \infty & \text{if such } n \text{ does not exist} \end{cases}.$$

We are interested in Δ_α - renewal function, which is defined as $R(t) = EN(t)$. We examine its properties in the Williamson transform terms.

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Estimation of the supremum of a class of sums of i.i.d. random variables

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Let a sequence of i.i.d. random variables $\xi_1, \dots, \xi_n$ be given together with a countable class of functions $\mathcal{F}$ such that $Ef(\xi_1) = 0$ for all $f \in \mathcal{F}$. We also assume that all functions $f \in \mathcal{F}$ are bounded by 1 in the supremum norm. Let us define the sums $S_n(f) = \sum_{j=1}^n f(\xi_j)$ for all $f \in \mathcal{F}$. We want to give a good estimate on the probability

$$P\left(\sup_{f \in \mathcal{F}} |S_n(f)| > x\right)$$

for all numbers $x > 0$.

It is worth considering this problem together with its natural Gaussian counterpart, where a countable set of normal random variables η_t , $E\eta_t = 0$, $t \in T$, is given, and we want to bound the probability $P\left(\sup_{t \in T} |\eta_t| > x\right)$ for all $x > 0$. There is a natural solution of this Gaussian problem with the help of the so-called chaining argument. To solve this problem we introduce the metric $d(\cdot, \cdot)$ on T , defined as $d^2(t, t') = E(\eta_t - \eta_{t'})^2$ for $t, t' \in T$. The estimate on the probability in the Gaussian problem depends on the cardinality of the smallest ε -nets with respect to this metric d in the metric space (T, d) , i.e. on the smallest number $n = n(\varepsilon)$ such that there are some points $t_1, \dots, t_n$ in T with the property that $\min_{1 \leq j \leq n} d(t, t_j) \leq \varepsilon$ for all $t \in T$. The estimate depends on the value of this quantity for all $\varepsilon > 0$.

Because of the central limit theorem it is natural to expect that our original problem has a solution similar to its Gaussian counterpart. But the situation is more complex. Some difficulties arise, since although we have a good Gaussian approximation of the probability $P(S_n(f) > x)$ for typical numbers x , but if the number $x > 0$ is very large in comparison to $Ef^2(\xi_1)$, then this probability behaves not so, as its Gaussian approximation would suggest. As a consequence, the methods worked out to solve the Gaussian problem do not give enough help to solve our original problem. To find a good estimate in this case we have to work out new methods, and in certain cases the solution of our problem may considerably differ from its Gaussian counterpart.

Our goal is to give a good solution to our problem. The first essential difference between this problem and its Gaussian counterpart is that our estimate depends on a Vapnik–Červonenkis distance type quantity, and not on the cardinality of certain ε -nets. K. Alexander [1] got an interesting result about this problem. He could tell when we have such an estimate that a comparison of our problem with its Gaussian counterpart suggests. But to get a complete picture that describes the situation also in the remaining cases a new method had to be worked out. I shall explain what is this new method, and what kind of results we get with its help.

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On natural convergence rate estimates in the Lindeberg–Feller theorem

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Let $X_1, \dots, X_n$ be independent random variables on a common probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with $\mathbf{E}X_k = 0$, $\mathbf{E}X_k^2 = \sigma_k^2$, and $B_n^2 = \sum_{k=1}^n \sigma_k^2 > 0$. Denote

$$\mu_k(z) = \mathbf{E}X_k^3 \mathbf{1}(|X_k| < z), \quad \lambda_k(z) = \mathbf{E}X_k^2 \mathbf{1}(|X_k| \geq z), \quad z > 0,$$

$$F_n(x) = \mathbf{P}(X_1 + \dots + X_n < xB_n), \quad \Delta_n = \sup_{x \in \mathbf{R}} |F_n(x) - \Phi(x)|,$$

where Φ stands for the standard normal distribution function. Let $\mathcal{G}$ denote the set of all nondecreasing functions $g: \mathbf{R}_+ \rightarrow \mathbf{R}_+$ such that $x/g(x)$ is nondecreasing for $x > 0$. We prove that for every $\varepsilon \in (0, \infty]$ and $\gamma \in (0, \infty)$ there exist constants $C_k = C_k(\varepsilon, \gamma)$, $k = 1, 2, 3, 4$, depending only on ε and γ such that

$$\begin{aligned} \Delta_n &\leq \frac{C_1}{B_n^3} \sup_{0 < z < \varepsilon B_n} \left(\gamma \left| \sum_{k=1}^n \mu_k(z) \right| + \sum_{k=1}^n \lambda_k(z) \right), \\ \Delta_n &\leq \frac{C_2}{B_n^3} \left(\gamma \left| \sum_{k=1}^n \mu_k(\varepsilon B_n) \right| + \sup_{0 < z < \varepsilon B_n} \sum_{k=1}^n \lambda_k(z) \right), \\ \Delta_n &\leq \frac{C_3}{B_n^2 g(B_n)} \sup_{0 < z < \varepsilon B_n} \frac{g(z)}{z} \left(\gamma \left| \sum_{k=1}^n \mu_k(z) \right| + \sum_{k=1}^n \lambda_k(z) \right) \quad \forall g \in \mathcal{G}, \\ \Delta_n &\leq \frac{C_4}{B_n^2 g(B_n)} \left(\frac{g(\varepsilon B_n)}{\varepsilon B_n} \cdot \gamma \left| \sum_{k=1}^n \mu_k(z) \right| + \sup_{0 < z < \varepsilon B_n} \frac{g(z)}{z} \sum_{k=1}^n \lambda_k(z) \right) \quad \forall g \in \mathcal{G}. \end{aligned}$$

The obtained inequalities improve and generalize those in [1]–[3] and, according to Zolotarev’s [4] classification, can be called *natural* convergence rate estimates in the Lindeberg–Feller theorem.

Moreover, we provide two-sided bounds for the constants C_k which are rather close to each other.

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Estimating parameters of a directed weighted graph model with beta-distributed edge-weights

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We introduce a directed, weighted random graph model, where the edge-weights are independent and beta-distributed with parameters depending on their endpoints. We will show that the row- and column-sums of the transformed edge-weight matrix are sufficient statistics for the parameters, and use the theory of exponential families to prove that the maximum likelihood estimate of the parameters exists and is unique. Then an algorithm to find this estimate is introduced together with convergence proof that uses properties of the digamma function. Simulation results and applications are also presented.

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Analysis of time series of amplitude-modulated plasma turbulence

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The L-2M stellarator for many years hosted experiments on high-temperature plasma turbulence [1]. The setup contains various physical diagnostics and a control system for electron cyclotron resonance heating (ECRH). The output of these plasma diagnostics is a set of time-series. These series contain information about evolution of plasma turbulence. New experimental conditions made it possible to investigate amplitude-modulated plasma turbulence [2,3].

Ability to determine one of the external parameters over time (plasma heating) allowed to clearly define the boundary of two different states of plasma turbulence. One area is characterised by the presence of heating radiation, and the second by its absence. Checks on the ergodicity in each of the areas of assessment of the evolution of the statistical moments and EM-analysis were conducted. The presence of transient processes allowed us to obtain new physical results: increase of the short-wavelength density fluctuations level while the long-wavelength density fluctuations level remains almost constant during the whole microwave pulse train. There are two regions of erasing turbulence: the edge and the centre of plasma volume.

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Stochastic correlations for modelling joint behaviour of asset prices

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To model correctly the joint behaviour of asset prices is particularly essential when pricing derivatives dependent on those assets. Typical examples are various spread options often written on two commodity prices, e.g. gas and oil, or quanto options protecting investors having exposure to a foreign asset written in foreign currency from the additional exposure to foreign exchange rates. The usual Pearson correlation is insufficient to represent properly asset interdependence because it is non-linear and has a temporal variation. Some authors suggest stochastic correlations, as explained below, and give approximate pricing for quanto options, with little if any justification based on the underlying. We assess this issue from the point of copulas and joint level exceedances. We suppose that the individual asset prices follow a Brownian motion driven model e.g. geometric BM or Heston or subordinated BM model. Hence, we build up *dependent* Wiener processes $W_t^{(i)}$ -s by modifying the simple one factor model. In the latter the driving Wiener processes admit a decomposition with a common Wiener process Z_t and idiosyncratic ones $\varepsilon_t^{(i)}$, expressed as

$$dW_t^{(i)} = \rho dZ_t + \sqrt{1 - \rho^2} d\varepsilon_t^{(i)}.$$

A constant ρ here can be interpreted as correlation among the Wiener components. Instead, we change it to be a random process ρ_t that we call *stochastic correlation*, in order to create a more complex non-linear interdependence structure. When ρ_t is adapted, and the idiosyncratic components are independent of Z_t and of each other, Lévy's characterisation theorem yields that all $W_t^{(i)}$ -s are Wiener processes. As a result, the asset prices generated from these Wiener processes have intact marginal distributions.

We suggest to create the stochastic correlation ρ_t either by a Jacobi process driven by the factor, or as a tangent hyperbolic transformation of a diffusion process. In either way the joint distribution of $W_t^{(i)}$ -s at fixed time t ceases to be multidimensional Gaussian. This is illustrated by Kendall's K-function in simulated processes. Furthermore formal tests reject the created copula to belong to known copula families. Some particular examples of high frequency stock prices will also be studied from this point of view.

On exact strong laws of large numbers under general dependence conditions

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The presentation will be based on the paper [1] where we study the almost sure convergence of weighted sums of dependent random variables to a positive and finite constant, in the case when the random variables have either mean zero or no mean at all.

These are not typical strong laws and they are called exact strong laws of large numbers. We do not assume any particular type of dependence and furthermore consider sequences which are not necessarily identically distributed. The obtained results may be applied to sequences of negatively associated random variables.

Some extensions to the random field setting will be also presented.

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Spatially adaptive Bayesian estimation for probabilistic temperature forecasts

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Weather forecasting today is based on deterministic numerical weather prediction (NWP) models. To represent model uncertainty, ensembles of NWP forecasts are employed, obtained by running the NWP model multiple times, each time with a different set of model conditions. However, forecast ensembles often exhibit systematic biases and dispersion errors. To improve the ensemble forecast skill and obtain reliable forecasts, statistical postprocessing models have been developed and applied with great success.

To account for dependencies in space, this work proposes a spatially adaptive extension of the state-of-the-art Ensemble Model Output Statistics (EMOS) model. The new approach, named Markovian EMOS (MEMOS), introduces a Markovian dependence structure on the model parameters by employing Gaussian Markov random fields. For fitting the MEMOS model in a Bayesian fashion the recently developed Integrated Nested Laplace Approximation (INLA) approach is utilized, allowing for fast and accurate approximation of the posterior distributions of the parameters. To obtain physically coherent forecasts the basic MEMOS model is provided with an additional spatial dependence structure induced by the Ensemble Copula Coupling (ECC) approach, which makes explicit use of the rank order structure of the raw ensemble.

The method is applied to 24-h temperature forecasts of the European Center for Medium-Range Weather Forecasts (ECMWF) over Germany, where it outperforms standard univariate EMOS variants in terms of several verification scores. When testing for equal forecast accuracy with the Diebold-Mariano test, it can be shown that this improvement is indeed statistically meaningful.

Bias reduction for the Hill estimator

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Estimating the tail index parameter is very important in extreme value theory. For heavy-tailed distributions the Hill estimator is simple and widely used for the problem above. This estimator is based on the k upper order statistics, and to find the optimal k is by far not trivial. Recent works aimed at the improvement of the Hill estimator by different methods, e.g. by using the double bootstrap [1], or the Kolmogorov-Smirnov metric [2]. These methods are asymptotically consistent, but for tail index $\xi > 0.5$ and smaller sample sizes the estimation fails to approach the theoretical value.

We realised that the method based on Kolmogorov-Smirnov distance results in biased estimators when the tail index $\xi > 0.5$ and the sample size is less than 10 000. Moreover the distribution of the estimator can be modelled by a generalized extreme value (GEV) distribution. We observed a linear relation between the location parameter of the fitted GEV distribution and the theoretical tail index in case of $(0.5 < \xi < 4)$.

Using the m out of n bootstrap technique for resampling one can calculate the Hill estimation of the bootstrap samples by the Kolmogorov-Smirnov method for any given sample. After linear transformation one can receive the corrected tail index estimation if $(0.5 < \xi < 4)$. This new algorithm is called the fitted regression method.

We compared the new regression estimator to the other tail index estimators. Our experience is that in the interval $(0.5 < \xi < 4)$ the new fitted regression method is better than the Kolmogorov-Smirnov-based one, and is similar to double bootstrap. However the computation time for the regression estimator is much less than for the double bootstrap.

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Performance simulation of finite-source cognitive radio networks with servers subject to breakdowns and repairs

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The present paper deals with the performance evaluation of a cognitive radio network by the help of a queueing model. The queueing system contains of two interconnected, not independent sub-systems. The first part is for the requests of the Primary Units (PU). The number of sources is finite and each source generates high priority requests after a generally distributed time. The requests are sent to a single server unit or Primary Channel Service (PCS) with preemptive priority queue. The service times are supposed to be also generally distributed. The second part is for the requests of the Secondary Units (SU). There are finite sources, too, the inter-request times and service times of the single server unit or Secondary Channel Service (SCS) are assumed to be generally distributed, respectively. A generated high priority packet goes to the primary service unit. If the unit is idle, the service of the packet begins immediately. If the server is busy with a high priority request, the packet joins the preemptive priority queue. When the unit is engaged with a request from SUs, the service is interrupted and the interrupted low priority task is sent back to the SCS. Depending on the state of secondary channel the interrupted job is directed to either the server or the orbit. In case of requests from SUs if the SCS is idle, the service starts, if the SCS is busy, the packet looks for the PCS. In case of an idle PCS, the service of the low priority packet begins at the high priority channel (PCS). If the PCS is busy the packet goes to the orbit. From the orbit it retries to be served after a generally distributed time.

The novelty of our investigation is that each server is subject to random breakdowns in which case the interrupted request is sent to the queue or orbit, respectively. The operating and repair times of the servers are supposed to be generally distributed. Finally, all the random times included in the model construction are assumed to be independent of each other.

The main aim of the paper is to analyze the effect of the non-reliability of the servers on the mean and variance of the response time for the SUs by using simulation.

Strong laws for weighted sums

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X_n , $n \geq 1$, are centered, strictly stationary and (positively) associated random variables. Let $a_{n,i}$, be nonnegative real numbers such that $A_\alpha = \sup_n n^{-1/\alpha} \left(\sum_{i=1}^n a_{n,i}^\alpha \right)^{1/\alpha} < \infty$, for some $\alpha > 1$. We will be interested in the almost sure convergence of the weighted sum $n^{-1/p} T_n = n^{-1/p} \sum_{i=1}^n a_{n,i} X_i$, for $p \in (0, 2)$. Define, for $i, j \geq 1$, $\Delta_{i,j}(u, v) = P(X_i > u, X_j > v) - P(X_i > u)P(X_j > v)$, $u, v \in \mathbb{R}$.

Theorem. Assume that $\sup_{i,j \geq 1} \Delta_{i,j}(u, v) = O(\max(|u|, |v|)^{-a})$, for some $a > 2$, as $\max(|u|, |v|) \rightarrow \infty$. Let $0 < p < 2$ and $\alpha > 1$ satisfy $\frac{1}{p} - \frac{1}{\alpha} = \frac{1}{2} + \xi$, for some $\xi \in (\frac{2}{a}, 1)$, and $\text{Cov}(X_1, X_n) \sim \rho^n$, for some $\rho \in (0, 1)$. If the weights satisfy $a_{n,i} \geq a_{n-1,i}$, then, with probability 1, $n^{-1/p} T_n \rightarrow 0$.

The assumption on $\Delta_{i,j}$ implies only the existence of moments up to order $[a]$. As an application, we derive the almost sure convergence of the weighted empirical distribution function.

For random variables without means, based on a suitable version of the Three Series Theorem, we prove the following result.

Theorem. Assume $\sup_{n \geq 1} E|X_n|^q < \infty$, for some $q \in (0, 1)$, and define $G_{i,j}(x, y) = \int \int_{[-x, x] \times [-y, y]} \Delta_{i,j}(u, v) du dv$. If

$$\sum_{n=1}^{\infty} \frac{1}{a_n^q} < \infty \quad \text{and} \quad \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \frac{1}{a_i a_j} G_{i,j}(a_i, a_j) < \infty,$$

then $\sum_n \frac{X_n}{a_n}$ is almost surely convergent. Moreover, if additionally the weights $a_n \nearrow \infty$, then $\frac{1}{a_n} (X_1 + \cdots + X_n) \rightarrow 0$ almost surely.

This allows for the relaxation of assumptions implying Marcinkiewicz-Zygmund strong laws.

Some results of the order of functions at infinity

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We define new classes of positive and measurable functions in terms of the asymptotic behaviour of its logarithm at infinity. Among others we study the class $\mathcal{M}(\rho)$ of functions satisfying $\log U(x)/\log x \rightarrow \rho$ as $x \rightarrow \infty$, where ρ may be finite or infinite. It is quite natural to look for other suitable normalizing functions than $\log x$. For a large class of functions $H(x)$ we study functions satisfying $\log U(x)/H(x) \rightarrow \rho$. These new classes of functions extend the class of regularly varying functions and the gamma class of function. In the paper we provide different characterizations of the new classes and we consider integrals, convolutions and Laplace transforms. We give some applications in probability theory. We also consider some natural extensions.

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Statistics for two-factor affine diffusions

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For every $a \in [0, \infty)$, $b, \alpha, \beta, \gamma \in (-\infty, \infty)$, $\sigma_1, \sigma_2, \sigma_3 \in [0, \infty)$ and $\varrho \in [-1, 1]$, the SDE

$$\begin{aligned} dY_t &= (a - bY_t) dt + \sigma_1 \sqrt{Y_t} dW_t, \\ dX_t &= (\alpha - \beta Y_t - \gamma X_t) dt + \sigma_2 \sqrt{Y_t} \left(\varrho dW_t + \sqrt{1 - \varrho^2} dB_t \right) + \sigma_3 dL_t, \end{aligned}$$

for $t \in [0, \infty)$ with an arbitrary initial value (Y_0, X_0) with $P(Y_0 \in [0, \infty)) = 1$ and independent of a 3-dimensional standard Wiener process $(W_t, B_t, L_t)_{t \in [0, \infty)}$, has a pathwise unique strong solution being a two-factor affine diffusion process. Conversely, every two-factor affine diffusion process is a pathwise unique strong solution of a SDE above with appropriate parameters $a \in [0, \infty)$, $b, \alpha, \beta, \gamma \in (-\infty, \infty)$, $\sigma_1, \sigma_2, \sigma_3 \in [0, \infty)$ and $\varrho \in [-1, 1]$.

First, sufficient conditions are presented for the existence of a unique stationary distribution and exponential ergodicity of two-factor affine diffusion processes, see [1].

Next, we study asymptotic properties of conditional least squares estimators for the drift parameters of two-factor affine diffusions based on continuous time observations. We distinguish three cases: subcritical, critical and supercritical. For all the drift parameters, in the subcritical and supercritical cases asymptotic normality and asymptotic mixed normality is proved, while in the critical case non-standard asymptotic behavior is described, see [2].

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The stochastic forecast model with the lead time 12–48h of the strong squalls, tornadoes and heavy precipitation over the territories of Europe and European part of Russia

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The development of successful method for automated statistical well-in-advance forecast (from 12 hours to two days) of the summer severe storm winds, squalls and tornadoes is very actual problem and is very difficult problem for synoptic operative practice. Nowadays there is no successful hydrodynamic model for the prediction of such phenomena (the wind velocity $V > 24\text{m/s}$ and the quantity of precipitation $Q > 25\text{mm/12h}$), hence the main tools for the objective forecast development are the methods using the stochastic model of these phenomena recognition [1]. The Byes approach was used for the development of the recognition discriminant functions of storm wind velocity and of heavy precipitation. The output production of regional model of Hydrometcenter of Russia i.e. the values of prognostic fields of this hydrodynamic model were used in our calculated discriminant function $F_1(X)$ and $F_2(X)$.

Prior to the recognition problem the problem of the selection of the most informative and slightly dependent parameters of atmosphere from 38 parameters was decided by the method of diagonalization of mean correlation matrix R . It was selected 6 parameters for the calculation of $F_1(X)$ and 7 parameters for the calculation of $F_2(X)$. These discriminant functions and the values of probabilities $P_1(X)$ and $P_2(X)$, depending from corresponding discriminant functions, are calculating automatic in the greed of the regional model $75 \times 75\text{km}$ at the operative system of Hydrometcenter of Russia two times per day with the lead time 12, 24, 36, 48h.

Thus the prognostic areas of these phenomena over the territories of Europe and Russia were shown on the map with the restricting by the isoline of the threshold probability P_{th} . These forecasts to 12–36 hours ahead for the territory of European part of Russia were recommended for the using in the operative practice of Hydrometcenter of Russia after the independent tests [2].

We are doing the verification of these forecasts for the different regions of Europe too. These stochastic forecast methods are successful enough in the Central and North Europe [3]. We are going to submit a lot of examples of such successful forecast for these territories in our report. This year the very strong squall ($V = 30\text{m/s}$) was observed over Moscow on 29.05.2017 and at next day 30.05.2017 the prognostic area of dangerous wind already was over Germany. In fact the wind velocity $V = 25\text{m/s}$ was observed in Berlin and other regions of Germany. The heavy and dangerous rainfalls were observed in Berlin too (the water was observed in the metro of Berlin). This and other examples of the forecast of storm wind and heavy precipitation with the lead time 12–48h over different regions of Europe will be

submitted at the seminar. The new developed technology allows take these maps from server of HMC to us and to other meteorologists. We are going to use this stochastic forecast model of these phenomena forecast for the development of the automated forecast to 60–72h ahead on the base of the global hydrodynamic model of Hydrometeorological Center of Russia (the author Tolstykh M.A.)

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Scaling transition for nonlinear random fields with long-range dependence

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[2], [3] introduced the notion of scaling transition for a stationary random field X on $\mathbb{Z}^2$, whose normalized partial sums over rectangles with sides growing at rates $O(n)$ and $O(n^\gamma)$ tend to an operator scaling random field V_γ on $\mathbb{R}^2$ for any $\gamma > 0$. The random field X is said to exhibit scaling transition if there exists a unique $\gamma_0 > 0$ such that the scaling limits V_γ are different and do not depend on γ for $\gamma > \gamma_0$ and $\gamma < \gamma_0$. In this talk I will present a complete description of anisotropic scaling limits and the existence of scaling transition for nonlinear functions (Appell polynomials) of a stationary linear random field on $\mathbb{Z}^2$ with moving average coefficients decaying at possibly different rate in the horizontal and vertical direction.

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Stationary distribution and perturbation bounds for a stochastic inventory model

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For the stochastic inventory problem, (s, S) policies are proved to be optimal under mild conditions. However, the computation of the optimal parameters is difficult. The analytical solution requires the computation of the renewal function of the demand distribution which is not available in closed form for general distributions. Furthermore, the demand distribution is generally unknown and is estimated from historical data via statistical methods. Hence, the mathematical model is perturbed by the approximations and estimation errors and this results in the deviation of the performance measures. It is important to ensure that those deviations remain limited. Otherwise, the mathematical model cannot be trusted and used as a reliable representation of the real system.

We consider the (s, S) inventory model with stochastic demand and backlogs. Under some conditions, the inventory position Markov chain is finite and aperiodic. It admits a unique stationary distribution.

Firstly, we will derive a formula to compute the stationary distribution from the transition matrix P of the considered inventory position Markov chain using a generalized inverse of $(I - P)$ (with I is the identity matrix). Then, we explore the effect of small perturbations of the demand distribution on the optimal inventory parameters and the performance measures of the system. We derive bounds on the deviation of the stationary characteristics of the considered inventory system and provide numerical examples to show the tightness of the derived perturbation bounds. In particular, we study the effect of estimation errors when the demand distribution is fit from historical data using the empirical distribution and moments matching methods.

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Weak invariance principle in Besov spaces for stationary martingale differences

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The classical Donsker weak invariance principle is extended to a Besov space framework. Polygonal line processes build from partial sums of stationary martingale differences as well from partial sums of independent and identically distributed random variables are considered. The results obtained are shown to be optimal. The talk is based on the paper [1].

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Optimal investment with general preferences in markets with frictions

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We consider a model of an illiquid market where trading faster leads to more unfavourable prices and the cost of illiquidity is superlinear in the trading speed. We consider an investor who, in the spirit of cumulative prospect theory, may have a non-concave utility function and may distort probabilities by exaggerating the likelihood of extreme events. The existence of optimal strategies for such agents is shown in great generality. Our main tool will be an extension of the well-known Skorohod representation theorem for sequences of weakly convergent random variables.

High-dimensional function-on-scalar regression in Hilbert spaces

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In this presentation I will discuss recent work concerning function-on-scalar regression when the number of predictors is much larger than the sample size. In particular, I will present a new methodology, called FLAME for Functional Linear Adaptive Mixed Estimation, which simultaneously selects, estimates, and smooths the important predictors in the model. Our methodology is readily available as an R package that utilizes a coordinate descent algorithm for fast implementation. Asymptotic theory will be provided and we will compare to previous methods via simulations.

Testing for change points in the cross-covariance of functional time series

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When considering two or more time series of functions or curves, for instance those derived from densely observed intraday stock price data of several companies, the empirical cross-covariance operator is of fundamental importance due to its role in functional lagged regression and exploratory data analysis. Despite its relevance, statistical procedures for measuring the significance of such estimators are undeveloped. We present methodology based on a functional central limit theorem for conducting statistical inference for the cross-covariance operator estimated between two stationary, weakly dependent, functional time series. Specifically, we consider testing the null hypothesis that two series possess a specified cross-covariance structure at a given lag. Since this test assumes that the series are jointly stationary, we also develop a change-point detection procedure to validate this assumption, which is of independent interest. The most imposing technical hurdle in implementing the proposed tests involves estimating the spectrum of a high dimensional spectral density operator at frequency zero. We propose a simple dimension reduction procedure based on functional PCA to achieve this, which is shown to perform well in a small simulation study. We illustrate the proposed methodology with an application to densely observed intraday price data of stocks listed on the NYSE.

A random graph model driven by time-dependent branching dynamics

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Consider an increasing group of individuals who are organized by pairwise collaborations. A successful collaboration attracts newcomers, who start collaborating with one or both participants. However, the new connections can weaken and exhaust the attracting pair's collaboration, which eventually ceases. Representing collaborations with edges, we get an evolving graph process.

To investigate the properties of this random graph model, it is a very natural idea to use the so-called Crump–Mode–Jagers branching process. However, our approach is different from the usual embedding scheme in the sense that edges, not vertices, are subject to branching. With this method, we could compute the probability that the graph becomes empty (extinction), deal with the asymptotics of the number of edges and vertices, furthermore, specify how the degree of a fixed vertex is evolving in time.

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BSDEs driven by a piecewise deterministic Markov process and featuring generalized Fréchet subgradients. Applications to multi-stable gene networks dynamics

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We investigate a mathematical model associated to the infection time in multistable gene networks. The mathematical processes are of hybrid switch type. The switch is governed by pure jump modes and linked to DNA bindings. The differential component follows backward stochastic dynamics (of PDMP type) and is reflected in some mode-dependent non-convex domains. First, we study the existence of solution to these backward stochastic variational inclusions (BSVI) by reducing them to a family of ordinary variational inclusions with generalized reflection in semiconvex domains. Second, by considering control-dependent drivers, we hint to some model-selection approach by embedding the (controlled) BSVI in a family of regular measures. Regularity, support and structure properties of these sets are given.

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Asymptotic degree distribution in preferential attachment graph models with multiple type edges

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Various types of random graphs with preferential attachment dynamics have been examined recently. The analysis of random graphs is motivated by the examination of large real networks, such as the internet and different kinds of biological and social networks. In some of the applications, the vertices and the edges may have different types. For example, in a social network, the vertices can be considered as males or females, and the edges can be considered as family or work relationships. This leads to different phenomena as if we assign weights to the vertices or to the edges.

In the current work, we extend the preferential attachment model by choosing the type of each edge randomly. We assume that there is a connection between the evolution of the structure of the graph and the types of the edges. In the n -type case, we define the (generalized) degree of a given vertex as $k = (k_1, k_2, \dots, k_n)$, where k_i is the number of type i edges connected to it. For the sake of simplicity, we only consider the 2-type case, but the results and the proofs can be easily generalized for the n -type case. The types are represented by colours, thus every edge can be either blue or red. We prove the existence of an almost sure asymptotic degree distribution for a general family of coloured preferential attachment graphs. That is, we show that for every fixed k and l , the proportion of vertices with exactly k blue and l edges tends to some random variable as the number of steps goes to infinity. We also provide recurrence equations for the asymptotic degree distribution. In addition to the main results, some applications are included in the talk. Joint work with Ágnes Backhausz.

On the random functional central limit theorems with almost sure convergence

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In this talk, we present functional random-sum central limit theorems with almost sure convergence for independent nonidentically distributed random variables. We consider the case where the summation random indices and partial sums are independent. In the past decade several authors (cf. [1] - [8]) have investigated the almost sure functional central limit theorems and related 'logarithmic' limit theorems for partial sums of independent random variables. We extend this theory to almost sure versions of the functional random-sum central limit theorems for subsequences.

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Some limit theorems for the risk estimator in the stabilized thresholding method of signal processing

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The thresholding techniques for the wavelet coefficients of the signal functions have become a popular denoising tool because of their simplicity, computational efficiency and the possibility to adapt to the functions with different amount of smoothness in different locations.

The observed noisy wavelet coefficients are defined as

$$Y_{jk} = \mu_{jk} + \epsilon_{jk}, \quad j = 0, \dots, J-1, \quad k = 0, \dots, 2^j - 1,$$

where μ_{jk} are "clean" coefficients and ϵ_{jk} are i.i.d. $N(0, \sigma^2)$ random variables (here j is the scale, and k is the shift in the wavelet decomposition).

To denoise the signal function authors of [1] proposed a stabilized hard thresholding method which avoids the main disadvantages of the popular soft and hard thresholding techniques. This method uses the threshold function

$$\hat{\mu}_{jk} = E[\rho_H(Y_{jk} + \lambda \xi_{jk}, T) | Y_{jk}],$$

where $\rho_H(y, T) = y \mathbf{I}(|y| > T)$ is the hard thresholding function, T is the threshold, λ is a stabilization parameter and ξ_{jk} are i.i.d. $N(0, 1)$ random variables independent of Y_{jk} .

The MSE of this method is

$$R_J(T) = \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} E(\hat{\mu}_{jk} - \mu_{jk})^2,$$

and

$$\hat{R}_J(T) = \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \left[(Y_{jk} - \hat{\mu}_{jk})^2 + 2\sigma^2 \frac{\partial}{\partial Y_{jk}} \hat{\mu}_{jk} - \sigma^2 \right]$$

is its unbiased estimator [1].

In [2] we prove the following limit theorems for $\hat{R}_J(T)$.

Theorem 1. Let f be defined on a finite interval and be uniformly Lipschitz regular with an exponent $\gamma > 0$. Then

$$P \left(\frac{\hat{R}_J(T_U) - R_J(T_U)}{\sigma^2 \sqrt{2^{J+1}}} < x \right) \Rightarrow \Phi(x) \quad \text{when } J \rightarrow \infty.$$

Theorem 2. Let $f \in L^2(R)$ be defined on a finite interval, then for any $\alpha > 1/2$

$$\frac{\widehat{R}_J(T_U) - R_J(T_U)}{2^{\alpha J}} \rightarrow 0 \text{ a. s. when } J \rightarrow \infty.$$

This theorems provide theoretical foundation for using the value of $\widehat{R}_J(T)$ as a measure of quality for the considered denoising method.

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A comparative study of robust and stable estimates of multivariate location

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This work is concerned with the comparative analysis of various robust estimates of location under the generalized normal, Student t - and Tukey's gross error model distributions both in the univariate and multivariate cases [1].

The chosen set of estimates comprises the sample mean, sample median, classical robust Maronna's, Huber's and Hampel's estimates, a low complexity two-stage estimate with rejection of outliers by the Tukey's boxplot rule at the first step with the subsequent use of the sample mean to the cleaned data and Meshalkin-Shurygin's stable estimates [2]. The estimate performance is evaluated by efficiency, bias and mean squared error.

For univariate distributions, the best results are exhibited by the Hampel's and Huber's estimates; the Meshalkin-Shurygin's and two-step estimates perform rather well too. In the multivariate case, we mostly focus on the estimate performance dependency on the data dimension: here, the Huber's and two-step estimates perform best.

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On the rate of convergence in the global CLT for Poisson–binomial and mixed Poisson random sums

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We provide moment-type estimates of the accuracy of the normal approximation to Poisson–binomial and Poisson random sums of independent identically distributed random variables in the mean metrics, including the uniform one. We also present convergence rate estimates of the mixed Poisson random sums to the mixtures of the normal laws, in particular, geometric random sums of centred random summands to the Laplace distribution, negative binomial random sums to the mixtures of the normal laws (with the decrease of the “success” probability p) and to the normal law (with the growth of the number of degrees of freedom r). It is interesting to note that in the latter case, the constructed estimates depend also on the parameter p which allows to obtain faster rates of convergence $\mathcal{O}(\sqrt{p/r} + r^{-1})$ than the classical one $\mathcal{O}(r^{-1/2})$, if $p \rightarrow 0$ as $r \rightarrow \infty$, without additional moment-type assumptions on the random summands.

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First exit time probability density forms for life table data

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Few years ago we have proposed [3] the following form $g(t)$ for the probability density of for a stochastic process hitting for the first time a barrier.

$$g(t) = \frac{2}{\sigma\sqrt{2\pi}} \frac{|H - tH'|}{\sqrt{t^3}} \exp \left[-\frac{[H(t)]^2}{2\sigma^2 t} \right]$$

for the applications in demography $H(t) = 1 - (bt)^c$ is the Health State at time t whereas for a complicated machine this is the Functional State. The parameters b, c are of particular importance with the exponent c to account for the curvature of the declining health state process. For $c = 1$ the Inverse Gaussian appear. An more general form of $H(t)$ appear in [1]. The application in human mortality data sets was not so successful in specific cases and a second order form approximation [2] was tested.

$$g(t) = \frac{2}{\sigma\sqrt{2\pi}} \left[\frac{|H - tH'|}{\sqrt{t^3}} + \frac{\sqrt{t^3}H''}{2|H - tH'|^2} - \frac{|H - tH'|}{\sqrt{t^3}} P_0(T < t) \right] \exp \left[-\frac{[H(t)]^2}{2\sigma^2 t} \right]$$

As the extra term $P_0(T < t)$ was not easy to evaluate in order to produce an easy to apply equation form, a simpler attempt was to select another approximation by inserting a parameter k and transforming the second order approximation to the form:

$$g(t) = \frac{2}{\sigma\sqrt{2\pi}} \left[\frac{|H - tH'|}{\sqrt{t^3}} - \frac{k\sqrt{t^3}H''}{2|H - tH'|} \right] \exp \left[-\frac{[H(t)]^2}{2\sigma^2 t} \right]$$

Furthermore we can arrive in a very interesting formula for the second order approximation by introducing the above selected $H(t) = 1 - (bt)^c$ in the last formula and applications to Life Table data sets are done.

$$g(t) = \frac{2}{\sigma\sqrt{2\pi}} \left[\frac{|l + (c-1)(bt)^c|}{\sqrt{t^3}} + \frac{k\sqrt{t^3}c(c-1)b^c t^{(c-2)}}{2|l + (c-1)(bt)^c|} \right] \exp \left[-\frac{[l - (bt)^c]^2}{2\sigma^2 t} \right]$$

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Self-similar random sets and fractals for generalized contractions

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The purpose of this paper is to present some fixed point and strict fixed point results for (singlevalued and multivalued) generalized contractions in non deterministic case.

Keywords: fixed point, random self-similar set, random fractal, generalized contraction.

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Fluctuations of Kendall random walks

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In [1] the authors constructed Markov chains generated by generalized convolutions.

We focus on the case of the Kendall convolution defined by

$$\delta_x \triangle_{\alpha} \delta_y := T_M(\rho^{\alpha} \tilde{\pi}_{2\alpha} + (1 - \rho^{\alpha}) \tilde{\delta}_1),$$

where $M = \max\{|x|, |y|\}$, $m = \min\{|x|, |y|\}$, $\rho = \frac{m}{M}$, $\tilde{\pi}_{2\alpha}(dx) = \alpha|x|^{-2\alpha-1} \mathbf{1}_{[1, \infty)}(|x|)dx$ and $\tilde{\delta}_1$ is symmetrization of δ_1 .

We consider barrier crossing problem for extremal Markov process $\{X_n : n \in \mathbb{N}_0\}$ of the Kendall type with the transition probabilities:

$$\mathbb{P}(X_n \in A | X_0 = x) = \delta_x \triangle_{\alpha} \nu^{\triangle_{\alpha} n}(A),$$

for all Borel sets $A \subset \mathbb{R}$.

We prove Wiener-Hopf factorization and Spitzer equality analogues for Kendall random walks in the Williamson transform terms. The talk is based on [3] which is a continuation of research from [2].

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Probabilistic analysis of a redundant repairable system with two service operations

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In the traditional mathematical theory of reliability, namely in redundancy theory, the replacement of the failed component by redundant unit was regarded as instantaneous, and the duration of repair of failed unit significantly small compared to its lifetime. In the most studies of reliability of the redundant systems, the replacement of the failed main unit by redundant unit is not considered as a separate, independent maintenance operation. In these conditions, the mean duration of downtime is really negligible.

As a matter of fact, nowadays it is important not only to estimate the mean downtime, but also the nature of its variation around mean value, the problem of a full probabilistic analysis of downtime (including replacement time) is very urgent. It has been raised natural necessity to construct and investigate maintenance models which consider two types of maintenance operations: 1) replacement of failed main unit by stand by one, and 2) repair of failed unit (both main and redundant one).

Simultaneously, duplex and triplex systems are most widespread in redundancy practice. In the presented paper we investigate one of such type triplex system.

Particularly a redundant system with two types of maintenance services, with single repair server and single replacement server is considered. Repair times are random variables. Mathematical model for dependability and performance analysis is constructed and investigated. Explicit solution for transient probabilities in terms of matrix exponential is obtained. The result of the research can be used for further analysis of the system to derive some important characteristics with different distribution functions of repair time.

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Recent results on finite source retrial queues with collisions

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Retrial queues have been widely used to model many problems arising in telephone switching systems, call centers, info-communication networks just to mention some. There is no queueing in these systems, that is if an arriving customer finds the server busy it goes into the orbit from where he makes random retrials to get service. Again, if at the moment of arrival the server is busy he returns to the orbit and the whole procedure repeats until the server is idle. If at the moment of arrival from the source the server is idle the service starts immediately. It is assumed that the customers arrive from a finite number of sources N .

The novelty of the investigations is that if the arriving customer finds the server busy he involves into collision with the customer under service and thus both enter into the orbit. Clearly, in this case the utilization of the server, the throughput of the system decrease, waiting time, response time, number of retrials and the number of customers in the systems increase. The structure of the mathematical model describing the system behavior depends on the distribution of the source times, service times and retrial times which are supposed to be independent random variables.

The aim of the presentation is to give an overview on the recent research carried out by the help of analytic, numeric, stochastic simulation, asymptotic methods and to summarize the most important results obtained on this topic. Special attention is devoted to the distribution of number of customers at the service facility, distribution of number of retrials and finally to the most complicated problem, namely to the distribution of waiting and response time. Several figures will illustrate the problems in question.

Estimating the covariance function of isotropic fields on the sphere

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Estimating the covariance function of a Gaussian isotropic random field $T(x)$ on the unit sphere $\mathbb{S}_2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}$ of the Euclidean space $\mathbb{R}^3$ has primary importance. We assume that an observation $\hat{T}(x)$, $x \in \mathbb{S}_2$ is given and estimate the covariance function by

$$\hat{\mathcal{C}}(\cos \gamma) = \int_{\mathbb{S}_2} \int_{C(x, \gamma)} \hat{T}(x) \hat{T}(x_\gamma(\varphi, x)) \frac{d\varphi}{2\pi} \frac{\Omega(dx)}{4\pi}. \quad (1)$$

where $\Omega(dx) = \sin \vartheta d\vartheta d\varphi$ is the Lebesgue element of the surface area on $\mathbb{S}_2$ and $C(x, \gamma)$ is the set of unit vectors $x_\gamma(\varphi, x)$ such that the central angle between x and $x_\gamma(\varphi, x)$ is γ . We show that $\hat{\mathcal{C}}(\cos \gamma)$ follows a Rosenblatt type distribution. Following results of Veillette and Taqqu the asymptotic distribution of the truncated version of the estimator $\hat{\mathcal{C}}(\cos \gamma)$ is also given. One can substitute the usual estimator of the spectrum by estimating the covariance function first then using the Gauss-Legendre quadrature for estimation of the spectrum as well. The problem of cosmic variance is also considered in accordance with simulations and real data. In practice the observations are given on a high resolution discretized sphere, for instance the Cosmic Microwave Background anisotropies (CMB) data are given on a pixel structure called HEALPix hence the estimator (1) is approximated in the high accuracy. This talk is based on the work [1].

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Comparison of two operation modes of finite-source retrial queueing systems with collisions and non-reliable server by using simulation

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In this paper a finite-source retrial queueing system with collision [1] of the customers is treated by the help of a simulation program. The server is not reliable [2], [3] breakdowns depend on whether the server is busy or idle. The random variables involved in the model construction are supposed to be totally independent. The novelty of the investigation is to compare the performance measures of these modes, and several sample results are shown in figures illustrating the difference of the working modes on the performance measures such as mean and variance of response time, mean and variance of number of customers in the system, mean and variance of sojourn time in the orbit, mean and variance of time a customer spent in service.

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Local asymptotic properties for Cox-Ingersoll-Ross process with discrete observations

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In this talk, we consider a one-dimensional Cox-Ingersoll-Ross (CIR) process whose drift coefficient depends on unknown parameters. Considering the process discretely observed at high frequency, we prove the local asymptotic normality property in the subcritical case, the local asymptotic quadraticity in the critical case, and the local asymptotic mixed normality property in the supercritical case. To obtain these results, we use the Malliavin calculus techniques developed recently for CIR process by Alòs et al. [1] together with the L^p -norm estimation for negative moments of the CIR process obtained by Bossy et al. [4] and Ben Alaya et al. [2].

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Maximum likelihood estimation for nonparametric denumerable Markov models

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Numerous results are available in the literature concerning maximum likelihood estimation for Markov models. The interest is primarily focused on parametric models due to their simplicity and their interest for applications. For finite state space Markov models in discrete time, results date back to [1]. In this talk, we study the nonparametric MLE for denumerable (homogeneous) Markov models. We obtain that the MLE is strongly consistent and asymptotically Gaussian (as a sequence of stochastic processes), given that the true model corresponds to a positive recurrent Markov chain. By means of the functional delta method and some elementary operator theory, we also develop an appropriate framework for obtaining the asymptotic distribution of the MLE of a Fréchet differentiable functional of the Markov kernel together with an explicit form of its asymptotic covariance kernel in many cases of interest. This includes the MLE of the stationary distribution, the fundamental kernel and the mean recurrence times of the Markov chain. In this study we also show how this framework can be used for asymptotic inference in semi-Markov models ([2], [3]). By using our proposed methodology we obtain the asymptotic distributions of the MLE of Fréchet differentiable functionals of the semi-Markov kernel corresponding to several characteristics of interest for a positive recurrent semi-Markov chain.

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Recovering information lost due to discretization

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Data for a statistical analysis are always given in a rounded form, so they contain both a random error and a rounding error. The rounding errors are especially serious and must not be ignored when the sample size is large. These days, huge data sets become more and more usual due to the rapid development of the computer technologies, therefore there is a growing interest to statistical analysis of rounded data, see, for example, [1]-[5] and references therein.

Rounding can arise as a result of limited sensitivity of measuring equipment or can be artificial, in the form of binning, for example, when the number of observations is so large that either registration or realization of the statistical procedure for they directly is difficult. In this work we consider situations when data for statistical analysis are given in a rounded form, and the rounding errors (the discretization step) are comparable or even greater than the measurement errors. We study possibilities to achieve accuracy much higher than the discretization step and to recover information lost due to rounding. The main tool for solving this problem is the use of additional measurement errors.

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Testing for stationarity of functional time series in the frequency domain

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Interest in functional time series has spiked in the recent past with papers covering both methodology and applications being published at a much increased pace. We contribute to the research in this area by proposing stationarity tests for functional time series based on frequency domain methods. Setting up the tests requires a delicate understanding of periodogram- and spectral density operators that are the functional counterparts of periodogram- and spectral density matrices in the multivariate world. Properties of the proposed statistics are derived both under the null hypothesis of stationary functional time series and under the smooth alternative of locally stationary functional time series. The methodology is theoretically justified through asymptotic results. Evidence from simulation studies and an application to annual temperature curves suggests that the tests work well in finite samples.

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On the Erdős-Turán law for random permutations with cycle weights

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Let S_n be a group of all permutations of the set X with n elements and $M(\sigma)$ be an order of permutation $\sigma \in S_n$ (as the element of this group). In [1], P. Erdős and P. Turán, in particular, showed that, for any $x \in R$, as $n \rightarrow \infty$

$$\mathbf{P} \left\{ \ln M(\sigma_n) \leq \frac{\ln^2 n}{2} + x \sqrt{\frac{\ln^3 n}{3}} \right\} \rightarrow \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-y^2/2} dy. \quad (1)$$

where random permutation σ_n is uniformly distributed on S_n . Note that, according to E. Landau [2], the order of permutations from S_n changes from 1 to $\exp((1 + \delta(n))\sqrt{n \ln n})$, where $\delta(n) \rightarrow 0$. In contrast, it follows from (1) that there is a sequence $\varepsilon(n) \downarrow 0$ such that permutation σ_n has an order from $\exp((1/2 - \varepsilon(n)) \ln^2 n)$ to $\exp((1/2 + \varepsilon(n)) \ln^2 n)$ with the probability tending to 1. Thus, this result from [1] was a surprise for the specialists. It is called now as the Erdős-Turán Law. The short review on the next results in this direction may be found in a note [3]. And one new result. Let a fixed permutation $\sigma \in S_n$ has C_i cycles with lengths $i \in N$. Consider a random permutation τ_n having distribution $\mathbf{P}\{\tau_n = \sigma\} = \theta_1^{C_1} \dots \theta_n^{C_n} / H(n)$ where $\theta_i \geq 0$ is called as a weight of cycles of τ_n having lengths $i \in N$, and $H(n)$ are the corresponding normalizing constants. Suppose that the sequence $\{H(n)/n!\}$ is RO -varying with lower index $\alpha > -1$ and $\theta_i \leq \theta < \infty$. Then

$$\mathbf{P} \left\{ \left(\ln M(\tau_n) - \sum_{i=1}^n \frac{\theta_i \ln i}{i} \right) \leq x \sqrt{\sum_{i=1}^n \frac{\theta_i \ln^2 i}{i}} \right\} \rightarrow \Phi(x) \quad (n \rightarrow \infty).$$

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Asymptotically optimal service stations arrangements for a parametric family of criteria

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In the communication, we discuss the impact of the asymptotically optimal service stations arrangements on the segment of a straight line. Assuming the service request is a random variable distributed on that segment, we chose an expectation of the distance between the point of the request and the nearest station to the power of s as an optimality criterion. (s is a natural number.) This criterion is more flexible than just an expectation of that distance; for example, by varying s , we can optimize the service of long-distant points. We show that in case of small influence zones we can provide an appropriate approximation of a stations optimal arrangement by putting that station into the center of its influence zone. We used this fact to find an asymptotical lower bound of a criterion value for optimal arrangements assuming the number of service stations tends to infinity. Finally, we suggested the algorithm of stations arrangement and proved that it provides an asymptotically second-order optimal arrangement which explains its practical value.

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Smoothed estimators for extremes of dependent circular data

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The aim of this paper is to show the possible applications of the peaks-over-threshold (GPD) approach to bivariate circular data. In our earlier work we have introduced a method for GPD modelling, where the known upper limit of the data was utilized, [2]. As the available data is discrete and we are interested in a continuous curve as estimated upper limit (or given high quantile), either a kernel type regression may be used or a parametric model might be fitted. There are applications e.g. from the fields of weather data [1], where the traditional extreme-value models and spline smoothing is applied.

An additional challenge is the dependence among subsequent observations, common in real-life applications. The theoretical results remain valid under mixing-type conditions, called $D(u_n)$. However, the bootstrap-based reliability estimators need to be adjusted.

In our presentation we compare the different available approaches, including a non-parametric method for estimating the pointwise maximum (called frontier in this setup, [3]). We estimate the reliability of the estimators by a suitable application of the m out of n bootstrap.

We illustrate the results for simulated as well as real data - principal components in the tangent space for enzymes data (88-dimensional time series). Here we were interested in determining the possible maximum and also the high quantiles for the 2-dimensional directions.

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Poster session

The causal relationship between land cultivated cereal and cereals yield in Algeria

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This paper seeks to investigate the causal relationship between land cultivated cereal in Algeria and cereals yield in Algeria. The empirical analysis starts by analyzing the time series properties of the data which is followed by examining the nature of causality among the variables. Algeria is from the largest importing countries of cereals. An increase cultivated cereal in Algeria increases cereals yield in Algeria. This study analyzes how change in real crude land cultivated cereal in Algeria affects the real cereals yield of Algeria positively. The empirical analysis involves testing the time series characteristics of the data series (stationary) using ADF test and running the pairwise Granger causality test based on Eviews software.

Keywords land cultivated cereal .cereals yield Cointegration Granger causality Algeria.

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On the convergence of moving average processes for dependent random variables

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Let $\{Y_i, -\infty < i < \infty\}$ be a doubly infinite sequence of identically distributed and ψ -mixing random variables with zero means and finite variance and $\{a_i, -\infty < i < \infty\}$ an absolutely summable sequence of real numbers. In this paper, we prove the complete moment convergence of $\{\sum_{k=1}^n \sum_{i=-\infty}^{\infty} a_{i+k} Y_i / n^{1/p}; n \geq 1\}$ under some suitable conditions.

Keywords: Complete moment convergence, moving average, ψ -mixing.

Statistical inference for Hermite-driven Ornstein-Uhlenbeck processes

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My poster will review two recent contributions [2,3] to the parameter estimation in Hermite-driven Ornstein-Uhlenbeck processes.

In the first part, we construct a drift estimator for the Ornstein-Uhlenbeck type process

$$dY_t = -\alpha Y_t dt + dZ_t^{q,H},$$

driven by a Hermite process $Z^{q,H}$ of any order $q \geq 1$ and Hurst parameter $H \in (\frac{1}{2}, 1)$. The estimator is given explicitly and allows numerical simulations. We prove strong consistency as well as a **non**-central limit theorem when $q \geq 2$ and $H \in (\frac{1}{2}, 1)$. The Gaussian case ($q = 1$) is apart: we show that either central or non-central limit theorems may arise depending on the value of self-similarity parameter. It solves an open question posed in [4].

In the second part, we study the statistical estimation in presence of periodicities. More specifically, we consider this time a Hermite Ornstein-Uhlenbeck processes of the form $dX_t = (L(t) - \alpha X_t)dt + dZ_t^{q,H}$, where $L(t)$ is a periodic and parametric function. Neither maximum likelihood estimator nor least square estimator can be applied for the drift parameter. We propose an estimator, and we show its almost sure convergence; moreover a **non**-central limit theorem is obtained for all order $q \geq 2$ and all value of the Hurst parameter $H \in (\frac{1}{2}, 1)$. We also investigate the case $q = 1$, and we solve an open problem left in [1].

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The effect of spatial intensity correlation on estimation of the Cox model

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Survival data analysis occasionally encounters a situation that some unknown risk factors affect survival times. In some applications, the survival data are spatially correlated. While Cox and frailty models cannot explain this effect and may result in misunderstanding analysis. In such cases, we can use spatial survival models. In this paper using a simulation study we show that by increasing the variance of spatial correlation among survival data, Cox and frailty models lead us to less accurate estimates. But spatial survival model showed a good performance with different intensity correlation levels and censoring precentages. Some simulation results are presented in Table 1.

Table 1: Estimate of β along Cox model with Gaussian covariogram

σ^2	Censor	Iteration								
		100			200			500		
		Est.	MAPB	MSE	Est.	MAPB	MSE	Est.	MAPB	MSE
1	20%	0.783	21.578	0.049	0.791	20.869	0.045	0.803	19.880	0.035
	40%	0.758	24.276	0.057	0.771	22.977	0.047	0.781	21.578	0.038
	60%	0.751	24.775	0.065	0.763	23.776	0.059	0.763	23.616	0.044
	80%	0.719	28.172	0.073	0.751	24.775	0.061	0.752	24.575	0.052
4	20%	0.548	45.255	0.073	0.553	44.555	0.069	0.568	43.257	0.061
	40%	0.531	46.753	0.083	0.539	45.954	0.073	0.541	44.954	0.063
	60%	0.529	47.153	0.092	0.532	46.453	0.078	0.534	46.753	0.071
	80%	0.498	50.650	0.103	0.501	49.950	0.803	0.511	48.951	0.079

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A Bayesian variance-stabilizing kernel density estimator using a conjugate prior

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It is known that a kernel-type estimator does not produce constant estimator variance over domains. To obtain a homoscedastic kernel regression estimator, Nishida and Kanazawa (2015) proposed a variance-stabilizing (VS) bandwidth for a local linear regression estimator. In this study, we show variance-stabilization is possible using the bayesian approach with a conjugate prior in the case of the kernel density estimator.

In literature, Gangopadhyay and Cheung (2002) derived the Bayes' estimator of the local variable bandwidth $h^*(x)$ for a univariate kernel density estimator by assuming the kernel function is Gaussian and the prior of h is the inverse gamma distribution, $\tau(h) = (2/\Gamma(\alpha)\beta^\alpha)(1/h^{2\alpha+1})\exp[-1/(\beta h^2)]$, where $\alpha > 0$, $\beta > 0$. The closed form is given by

$$h^*(x) = \frac{\Gamma(\alpha)}{\sqrt{2\beta}\Gamma(\alpha + \frac{1}{2})} \frac{\sum_{i=1}^n \left[\frac{1}{\beta(X_i - x)^2 + 2} \right]^\alpha}{\sum_{i=1}^n \left[\frac{1}{\beta(X_i - x)^2 + 2} \right]^{\alpha + \frac{1}{2}}}. \quad (1)$$

To estimate a bayesian VS bandwidth for a kernel density estimator, the hyper parameters α and β in (1) should be determined to satisfy $h^*(x) = f_X(x)C$ at every x in the domain, where $f_X(x)$ is a true density function and C is a positive constant, satisfying the condition $\alpha > 0$ and $\beta \rightarrow \infty$, $n\beta^{-\frac{1}{2}} \rightarrow \infty$ as $n \rightarrow \infty$ in Kulasekera and Gallagher (2010). In the presentation, we show results about the hyperparameters and provide simulation.

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A comparison of two interaction based random graph models

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Random graphs (RGs) are widely used for modeling real world phenomena (see e.g. [1]). In the present work we study two dynamically evolving RG models. The one is introduced in [2] and generalized in [3]. Here the evolving step is based on adding a fixed size, appropriately chosen clique (a complete graph) to the network, that is, the selected nodes interact in a symmetrical, uniform way. The other is proposed by [4], which is based on extending the graph with a fixed size star (an N vertex tree with a central, $N - 1$ degree node), i.e., the interaction is highly skewed, concentrated. These models combine the uniform and preferential attachment (PA) selection similarly as in [5], except that, this process is based on a special weight structure attached to the network - the degree structure is less involved. In both cases the evolving step is controlled by three real parameters P, Q, R - which are accountable for the extension and selection type. The following common network measures and their relations to the model parameters will be empirically investigated: degree and weight distribution of vertices, clustering coefficient, centrality, diameter and the robustness through the method proposed in [6].

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General theorems on exponential and Rosenthal's inequalities and on complete convergence

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For a sequence of r.v.'s $\eta_1, \eta_2, \dots, \eta_n$ we shall consider the condition

$$\mathbb{E}e^{\sum_{i=1}^n \lambda \eta_i} \leq g(n) \prod_{i=1}^n \mathbb{E}e^{\lambda \eta_i} \quad (1)$$

where $0 < g(n) < \infty$. If condition (1) is satisfied for all $\lambda \in \mathbb{R}$ and for all n , then the sequence $\eta_1, \eta_2, \dots$ is called widely acceptable, see [1]. If $g(n) \equiv 1$, then we are at the notion of acceptable r.v.'s [2]. Exponential inequalities are obtained if (1) is satisfied for the truncated r.v.'s. Then it is shown that an exponential inequality implies both Rosenthal's inequality and complete convergence of sums of random variables. The general results are applied to weakly orthant dependent sequences.

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A new network evolution model based on co-operations

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Several real-life networks have a common property, namely they have power-law degree distribution. They are called scale-free networks.

Barabási and Albert [1] introduced the preferential attachment evolution model. This model leads to scale-free networks.

We introduce a new network evolution model which is based on co-operations. The choice of the cooperating individuals can be based either on the preferential attachment rule or can be made uniformly at random. The basic unit of the cooperation is a group of N individuals so that one of them plays central role and the others join to him/her. At each step N individuals co-operate which means that they form a directed N -star graph.

We present the asymptotic power-law distribution of the in-degrees and the out-degrees of vertices and the asymptotic power-law distribution of the weights of N -stars.

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The asymptotic behaviour of an N -interactions random graph model

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In the last two decades, examination of real-world networks became one of the most popular research topics. It turned out that the main common feature of the most of real-life networks is their scale-free nature. Networks are called scale-free when their asymptotic degree distributions follow power laws ([1]). A large number of random graph models has been proposed to describe complex networks. In this paper we give the description of a general random graph model evolving in discrete time steps. The evolution mechanism of the graph is a combination of the preferential attachment rule and the uniform choice of vertices. In our model every vertex is characterized by its degree and its weight. Moreover, the M -cliques ($1 \leq M \leq N$ fixed) also have weights. Asymptotic results for the model are presented. Scale-free weight distributions for the M -cliques are obtained. Power law degree distribution for several special cases of the general model is also proved.

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A chaotic approach for animal action recognition

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Animal activity is a descriptor that can be potentially used to track the health and well-being of individuals which is obviously very important to improve the management process and productivity of farms. The paper deals with an animal behaviour recognition problem using a chaos theory approach. In the framework of state recognition problem this theory was implemented for systematic characterisation of the electromyogram [1]. We intend to adopt such a technique for automatic classification of a calfs behaviour. Three main mutually exclusive states are of our interest: laying, standing and walking. The time series generated by active RFID sensors with a 3D-accelerometer after a denoising transformation and frequency stabilization are treated as representation of a non-linear dynamical system. This data material is used to construct attractors as subsets of phase-space characterized by dynamic and metric invariants including embedding delay, embedding dimension, Lyapunov exponent and correlation dimension. Each state is described by a feature vector which is represented as a combination of these invariants. To train the classifier, video-recorded observations synchronized with acceleration data sets are used. It is shown that the chaos theory approach can be used as alternative methodology for the animal behaviour state recognition problem with acceptable classification accuracy and as element of ensemble method in machine learning where a combination of different models is used.

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Baum–Katz type theorems with exact threshold

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For a sequence of random variables $\{X_n\}_{n \geq 1}$ and some positive parameters r, p consider the statement

$$\sum_{n=1}^{\infty} n^{p/r-2} P(M_n > \varepsilon n^{1/r}) < \infty \quad \text{for all } \varepsilon > 0, \quad (1)$$

and the weaker claim

$$\sum_{n=1}^{\infty} n^{p/r-2} P(|S_n| > \varepsilon n^{1/r}) < \infty \quad \text{for all } \varepsilon > 0, \quad (2)$$

where $S_n = \sum_{i=1}^n X_i$ and $M_n = \max_{1 \leq i \leq n} |S_i|$.

The goal of the paper is to investigate statements (1) and (2) for arbitrary random variables, martingale difference sequences, and centered sequences with a certain level of negative dependence. We will deduce (1) by assuming only a uniform moment condition $\sup_{n \geq 1} E(|X_n|^q f(|X_n|)) < \infty$, where the function $f: [0, \infty) \rightarrow (0, \infty)$ might be any non-decreasing function satisfying $\sum_{n=1}^{\infty} 1/f(2^n) < \infty$. We will find the smallest possible suitable constant $q = q(p, r)$ which we call the *critical exponent*. This leads to Marcinkiewicz–Zygmund type strong laws of large numbers with estimate for the rate of convergence.

Statistical inferences and optimal designs based on gamma processes

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During the past two decades, degradation analysis has been widely used to assess the life-time information of highly reliable products. Usually random effect models and/or Wiener processes are well suited for modelling the stochastic degradation. But in many situations, such as materials that lead to fatigue data, it is more appropriate to model the data by a gamma processes which exhibits a monotonic increasing pattern. This article surveys the theoretical aspects as well as the application of gamma processes in degradation analysis. Some statistical properties of degradation models based on gamma processes under different tests are given. Furthermore, the corresponding optimal designs for conducting the degradation experiments efficiently are reviewed. Finally, some extensions and applications of gamma-process degradation model are presented.

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Stability problems in Neumann-Pearson theorem and its application

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In [1,2] a problem of a state definition for an alternating process state by an observation of Poisson type point process with intensity controlled by the alternating process is considered. Its solution is based on methods of statistical hypothesis checking for parameters of point process provided its observation belongs to Poisson parametric distribution family approximately.

Consequently in a procedure of Bayes decision rule it is necessary to include approximations, to estimate inaccuracy and to insert this inaccuracy into known statistical procedure. Particularly using Neumann-Pearson theorem it is necessary to find not optimal but ε -optimal Bayes decision rule.

In this paper such estimates are constructed in an assumption observation time is not large. Results of some calculation experiment are represented.

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Consistent variance estimation based on rounded data

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It was shown in [1] how to construct a consistent estimator of the expectation, when the data are given in a rounded form, and the discretization step does not converge to zero. In this work, we consider the same problem for estimating the variance. Below we present one of the results obtained.

For a real number x let x^* denote the nearest integer. Suppose that $X_1, X_2, \dots$ are iid random variables with the second moment μ_2 which is estimated. The data are given in a rounded form, and there is the following possibility. One can generate iid random variables $\xi_1, \xi_2, \dots$ and observe variables $(X_1 + \xi_1)^*, (X_2 + \xi_2)^*, \dots$. How to choose the distribution of variables ξ -s in order to obtain a consistent estimator of μ_2 ? Denote $\mu_1 = EX_i$. The following inequality holds

Theorem Let ξ -s be normally distributed with zero mean and variance σ^2 . Then

$$\left| \lim_{n \rightarrow \infty} (a.s.) \frac{1}{n} \sum_{i=1}^n ((X_i + \xi_i)^*)^2 - \left(\mu_2 + \sigma^2 + \frac{1}{12} \right) \right| \leq \frac{1}{\pi} e^{-2\pi^2 \sigma^2} \left(2\mu_1 + 4\pi\sigma^2 + \frac{\mu_1 + 2\pi\sigma^2}{2\pi^2\sigma^2} + \frac{1}{\pi} + \frac{1}{4\pi^3\sigma^2} \right).$$

The theorem implies in particular that

$$\lim_{\sigma \rightarrow \infty} \left| \lim_{n \rightarrow \infty} (a.s.) \frac{1}{n} \sum_{i=1}^n ((X_i + \xi_i)^*)^2 - \left(\mu_2 + \sigma^2 + \frac{1}{12} \right) \right| = 0.$$

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Analysis of a single-server queue with phase-type service and reneging

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In classical queuing models it is assumed that customers are willing to wait until they get serviced. However in real life, customers may lose their patience due to the long service times or long queues and leave the system without having service. In this study, a single server queuing system with phase-type service is examined. Service time is assumed to have k -phase Cox distribution whereas arrivals are from Poisson process with a constant rate. When the server is busy upon an arrival, customer joins the queue and the queue capacity is infinite. Since the variance of the service time is relatively high, the service time distribution is characterized by k -phase Cox distribution. Due to the high variability of service times and since some of the service times are extremely long, the customers not only in the queue, but also in the service may become impatient. Each customer, upon arrival, activates an individual timer and starts his patience time. The patience time for each customer is a random variable which has exponential distribution with parameter ξ . If the service does not completed before the customer's time expires, the customer abandons the queue never to return.

The analysis of single-server queuing system with k -phase Coxian service by probability generating functions is examined in detailed. $M/C_k/1$ queuing model with reneging is expressed by using birth-and-death process and the balance equations are provided. In addition, expected number of customers in the system is derived by using probability generating function and some numerical results are presented.

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Joint analysis of current count and current status data

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We consider joint analysis of times to a recurrent and a non-recurrent event, with both events are examined only once and thus they produce current count and current status data respectively. The motivation arises from a survey study, which collected current count data for time to occurrences of fracture (recurrent event) and current status data for time to osteoporosis (non-recurrent event) for each participant. The aim of the study is to identify risk factors for fracture and osteoporosis. We propose a conditional likelihood analysis of current count and current status data under a joint frailty model where we assume a non-homogeneous Poisson process for the recurrent event, a proportional odds model for the failure time of the non-recurrent event, but leave the distribution of the common frailty shared by two event times processes unspecified. The key of the proposal is a sufficient statistic of the frailty can be found so that the derived conditional likelihood is independent of the frailty, and hence the distribution assumption on the frailty is unnecessary. Asymptotic properties of semiparametric estimators for parametric (covariate effects) and nonparametric (baseline mean and odds functions) parameters are derived and an efficient computation method is established. We illustrate the performance of the proposal via simulation studies and analysis of fracture-osteoporosis survey data.