Set, multiset and array

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- no repeated objects
- not necessarily objects of the same type
- without any particular order
- usually denoted by capital letter: A, B

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Examples:

1. $A = \{1, 2, 3\}$
2. $B = \{apple, -4, moon\}$
3. $C = \mathbb{N}$

- Let A be a set and a an object. If A contains a, then we write $a \in A$.
- If A does not contain a, then we write $a \notin A$.

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Example: $A = \{2, 4, 6, 8\}$

- $2 \in A$
- $3 \notin A$

Operations

- union: $A \cup B$ contains all elements which are either element in A or in B
- intersection: $A \cap B$ contains all elements which are element in A and B
- difference: $A \setminus B$ contains all elements of A, which are not element in B

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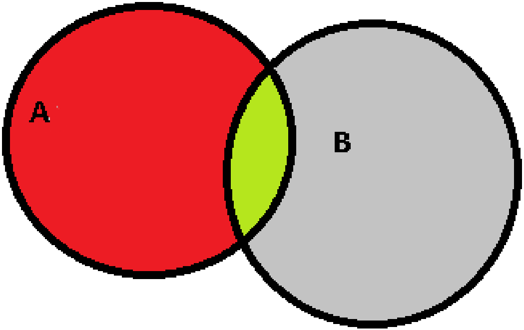

Venn diagram

Exercises

1. $A = \{-1, 0, 3, 5\}$, $B = \{1, 2, 3, 4, 7, 12\}$. Determine $A \setminus B$!
2. $A = \{p \mid p \text{ prime and } p \leq 10\}$, $B = \{n \mid n \text{ is even}\}$. Find $A \cup B$, $A \cap B$!
3. Verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$!

Multiset

- Allows repeated elements
- Each element has multiplicity
- Example: $\{1, 1, 2, 3, 3\}$ Then $m(1) = 2, m(2) = 1, m(3) = 2$

Characteristic Function

The characteristic function is an indicator function which show if an element is contained in a set or not.

$$F(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \notin S \end{cases}$$

- Union: $x \in A \cup B \Leftrightarrow x \in A \vee x \in B$
- Intersection: $x \in A \cap B \Leftrightarrow x \in A \wedge x \in B$
- Difference: $x \in A \setminus B \Leftrightarrow x \in A \wedge x \notin B$

Example

- Repition of elements is possible
- Example: $A = \{1, 1, 1, 2, 3, 3, 4, 5, 7, 7\}$
- Characterstic function:

	1	2	3	4	5	7
A	3	1	2	1	1	2

Exercise

1. $A = \{1, 1, 2, 4, 6, 8, 8, 8\}$, $B = \{2, 2, 2, 4, 4, 4, 5, 5, 5, 9\}$
2. $A = \{1, 5, 4, 7\}$, $B = \{0, 2, 4, 8, 7\}$

Fill out the characteristic function of the following multisets:

	1	2	4	5	6	8	9
A							
B							
$A \cup B$							
$A \cap B$							
$A \setminus B$							

Array - matrix

- k rows and n columns: $k \times n$ matrix
- diagonal matrix: nonzero elements only for $a_{i,i}$
- lower triangular matrix: nonzero elements only under the diagonal
- upper triangular matrix: nonzero elements only above the diagonal
- symmetric matrix: columns $=$ rows

Matrix - two dimensional array

- $M[r, c]$
- $M[i, j] = V[(i - 1) * c + j]$ Row Major Order Representation
- $M[i, j] = V[(j - 1) * r + j]$ Column Major Order Representation
- $j > i \Rightarrow M[i, j] = 0$ Lower triangular matrix
- $i > j \Rightarrow M[i, j] = 0$ Upper triangular matrix
- $M[i, j] = M[j, i]$ symmetric matrix

Exercise - Row major order representation

Matrix M is given in V vector as below: $V = [6, 76, 20, 20, 51, 88, 84, 47, 74, 46, 53, 22, 41, 88, 44, 1, 4, 95, 12, 55, 90, 11, 91, 62, 62, 33, 93, 88]$

1. Compute the value of $M[1,1] - M[2,4]$ if M is row major order represented, which has 7 rows and 4 columns!
2. Compute $M[1,2] + M[7,1] \mod 5$!

Column major order representation

Vector represents elements in first column, then second column, and so on.

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Compute the exercise again, but now M is column major order represented. What is the difference?

Sparse matrix

- Most elements are 0
- 3 row representation: row index, column index, element (for nonzero elements)

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$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 3 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \end{pmatrix}$$

ROW: (1,1,2,3)
COLUMN: (3,6,1,4)
VALUE: (1,3,-1,5)

Exercises

$V = [1, 7, 6, 5, 3, 8, 7, 2, 3, 4, -4, -6, 3, 2, -9, 1]$
Compute $M[2, 2] + M[3, 2]$ if M is a ... matrix represented by V

1. upper triangular matrix
2. lower triangular matrix
3. symmetric 4×4 matrix

Give the sparse matrix defined by ROW: (1,5,5,5,6,7)
COLUMN: (2,3,4,5,6,1)
VALUE: (-1,-1,4,7,-2,3)