Minor Field of Comprehensive Examination

Computational optimization

Syllabus

1. Uniform random number generators (congruential and shift register generators, the KISS generator). Transformation based generators, examples. Box-Müller algorithm. Accept-Reject and Envelope Accept-Reject algorithms, examples. Adaptive Rejection Sampling (ARS) algorithms.
2. Metropolis-Hastings algorithm, definition and invariant distribution. Convergence properties, recurrence, irreducibility and aperiodicity. Examples. Independent Metropolis-Hastings algorithm, geometric and uniform ergodicity. Examples. Adaptive Rejection Metropolis Sampling (ARMS) algorithm. Random walk Metropolis-Hastings algorithm, ergodicity. Examples.
3. Slice Sampler algorithm, definition, heuristics behind, convergence properties. Examples. Two-stage Gibbs Sampler, definition and Markov properties. Examples. Hammersley-Clifford theorem, recurrence, ergodicity, reversibility.
4. Solution of system of linear equations, direct and iterative methods (LU, PLU, Cholesky factorization, QR factorization, Householder transformation, Gauss and Gauss-Seidel iterations, relaxation methods) Singular value decomposition, pseudo-inverse.
5. Function approximations. Lagrange, Hermite and spline interpolations. Orthogonal polynomials. Fast Fourier transformation. Best uniform approximation. Least squares approximation.
6. Implementation techniques of simplex method and it variants, special technologies for large problems, methods for increasing computational efficiency. Modelling languages MPS, AMPL. Standard test problem libraries.
7. Unconstrained optimization problems. Optimality conditions. Convexity. Constrained optimization problems. Karush-Kuhn-Tucker theory. Line search

methods. Descent direction, step-size rules. Newton method. Quasi-Newton methods.

Bibliography

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3. Brian D. Ripley: *Stochastic Simulation.* Wiley, New York, 1987.
4. Sean Meyn, Richard Tweedie: *Markov Chains and Stochastic Stability.* Springer, New York, 1993.
5. J. Stoer, R. Bulirsch: *Introduction to Numerical Analysis*, Springer, 2002.
6. G.H. Golub, C.F. Van Loan: *Matrix computations*, John Hopkins Univ. Press, 1996
7. Jorge Nocedal and Stephen Wright: *Numerical optimization*, Springer, 2006
8. Osman Güler: *Foundations of optimization*, Springer, 2010.
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10. Maros István: *Computational techniques of the simplex method,* Springer, 2002.
11. Panos M. Pardalos, Mauricio G. C. Resende: *Handbook of applied optimization* (Part THREE – Software), Kluwer Acad. Publ., 2001