Matematika mérnököknek I.

Komplex számok

1. (a) $5+14i$
   (b) $20+10i$
   (c) $15+10i$
   (d) -10
   (e) $-9-6i$
   (f) $21-20i$
   (g) $-11+2i$
   (h) i
   (i) -2
   (j) $1-3i$
   (k) $\frac{4}{13}+\frac{19}{13}i$
   (l) $\frac{7}{10}-\frac{11}{10}i$

2. (a) $5(\cos\pi+i\sin\pi)$
   (b) $8(\cos 0+i\sin 0)$
   (c) $\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}$
   (d) $2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$
   (e) $\frac{\sqrt{2}}{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
   (f) $\sqrt{10}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$
   (g) $\frac{1}{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$
   (h) $4\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

3. (a) $8(\cos\pi+i\sin\pi)=-8$
   (b) $\frac{1}{2}\left(\cos\frac{9\pi}{5}+i\sin\frac{9\pi}{5}\right)$
   (c) $256\left(\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right)$
   (d) $8\left(\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right)$
   (e) $\frac{1}{2}\left(\cos\frac{7\pi}{5}+i\sin\frac{7\pi}{5}\right)$
   (f) $256\left(\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}\right)$

4. (a) $2^{153}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=2^{153}i$
   (b) $2^{54}(\cos\pi+i\sin\pi)=-2^{54}$

5. (a)

$$\zeta_1=3\sqrt{3}\left(\cos\frac{3\pi}{10}+i\sin\frac{3\pi}{10}\right)$$
$$\zeta_2=3\sqrt{3}\left(\cos\frac{13\pi}{10}+i\sin\frac{13\pi}{10}\right)$$

(b)

$$\zeta_1 = 3\left(\cos\frac{3\pi}{15} + i\sin\frac{3\pi}{15}\right)$$
$$\zeta_2 = 3\left(\cos\frac{13\pi}{15} + i\sin\frac{13\pi}{15}\right)$$
$$\zeta_3 = 3\left(\cos\frac{23\pi}{15} + i\sin\frac{23\pi}{15}\right)$$

(c)

$$\zeta_1 = \sqrt[4]{27}\left(\cos\frac{3\pi}{20} + i\sin\frac{3\pi}{20}\right)$$
$$\zeta_2 = \sqrt[4]{27}\left(\cos\frac{13\pi}{20} + i\sin\frac{13\pi}{20}\right)$$
$$\zeta_3 = \sqrt[4]{27}\left(\cos\frac{23\pi}{20} + i\sin\frac{23\pi}{20}\right)$$
$$\zeta_4 = \sqrt[4]{27}\left(\cos\frac{33\pi}{20} + i\sin\frac{33\pi}{20}\right)$$

6. A negyedik gyökök:

$$\zeta_1 = \cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}$$
$$\zeta_2 = \cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}$$
$$\zeta_3 = \cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}$$
$$\zeta_4 = \cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}$$

Az ötödik gyökök:

$$\zeta_1 = \cos\frac{3\pi}{10} + i\sin\frac{3\pi}{10}$$
$$\zeta_2 = \cos\frac{7\pi}{10} + i\sin\frac{7\pi}{10}$$
$$\zeta_3 = \cos\frac{11\pi}{10} + i\sin\frac{11\pi}{10}$$
$$\zeta_4 = \cos\frac{15\pi}{10} + i\sin\frac{15\pi}{10}$$
$$\zeta_5 = \cos\frac{19\pi}{10} + i\sin\frac{19\pi}{10}$$

7. (a) $e^{i\frac{3\pi}{5}}$ (b) $2e^{i\frac{4\pi}{7}}$

8. (a) $z = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$
   (b) $z = 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$
   (c) $z = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -1 + \sqrt{3}i$
   (d) $z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

9.

$$zw = 8e^{i\frac{7\pi}{12}}, \quad \frac{z}{w} = 2e^{-i\frac{\pi}{12}}, \quad z^4 = 256e^{i\pi}, \quad z^2w^3 = 128e^{i\frac{3\pi}{2}},$$