Integration – homework

Exercise 1.

$$\int 5x^4 + 2x^3 - 6x^2 + x + 20\,dx \qquad \int \cos y + \sin y\,dy \qquad \int \frac{t^3 - t^2 + 1}{t}\,dt$$

$$\int \frac{x^2 + x - 1}{x^5}\,dx \qquad \int \sqrt{x} + \frac{1}{\sqrt{x}}\,dx \qquad \int 8^x\,dx$$

Exercise 2.

$$\int e^{3-8x}\,dx \qquad \int 2^{5x-1}\,dx \qquad \int \frac{1}{\cos^2(2x+3)}\,dx \qquad \int \sqrt[3]{7+4t}\,dt \qquad \int \frac{1}{1+(2-3x)^2}\,dx$$

$$\int \frac{1}{(7x-5)^3}\,dx \qquad \int 3\cdot e^{-2x}\,dx \qquad \int \cos(6x-4)\,dx \qquad \int \frac{7}{2-3x}\,dx \qquad \int e^{\frac{x}{5}}\,dx$$

Exercise 3.

$$\int \frac{1}{x\cdot \ln x}\,dx \qquad \int \cot x\,dx \qquad \int \frac{x-2}{x(x-4)}\,dx \qquad \int \frac{x}{(1+x^2)^2}\,dx$$

$$\int \frac{x}{(8x^2+27)^{\frac{2}{3}}}\,dx \qquad \int \frac{4x}{\sqrt[3]{x^2+6}}\,dx \qquad \int 2\cos^4 x\cdot \sin x\,dx \qquad \int \cos x\cdot \sin x\,dx$$

Exercise 4.

$$\int x^2\cdot \sin x\,dx \qquad \int (x+8)\cdot \cos(2x-1)\,dx \qquad \int (2x^4 - 3x^2 + x - 8)\cdot \ln(2x)\,dx$$

$$\int 5x\cdot e^x\,dx \qquad \int (x^2-2x+2)\cdot \cos x\,dx \qquad \int (x^2+2x-1)\cdot \ln x\,dx \qquad \int x\cdot \sin(3x)\,dx$$

Exercise 5.

$$\int \sqrt{x}\cdot e^{\sqrt{x}}\,dx \qquad \int \frac{x^2}{(2x+4)^4}\,dx \qquad \int \tan^2 x\,dx \qquad \int \frac{e^x}{e^{2x}+2e^x+1}\,dx$$

Exercise 6.

$$\int_{-\frac{\pi}{2}}^{\pi} 3\cos x\,dx \qquad \int_{-1}^{2} y^3 - 6y\,dy \qquad \int_{2}^{3} \frac{6}{x}\,dx \qquad \int_{0}^{4} \frac{2x}{x^2+1}\,dx$$

$$\int_{-2}^{2} \frac{2x}{(x^2-100)^7}\,dx \qquad \int_{-5}^{5} |x|\cdot e^x\,dx \qquad \int_{1}^{8} \sqrt[3]{x} - \frac{1}{2\sqrt[3]{x}}\,dx$$

Hints:

Exercise 2. Use the rule

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c,$$

where $\int f(x)\,dx = F(x) + c$.

Exercise 3. Use the rules

$$\int f^n(x)\cdot f'(x)\,dx = \frac{f^{n+1}}{n+1} + c \quad (\text{if } n \neq -1), \quad \text{and} \quad \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c.$$

Exercise 4. *Integration by parts.*

Exercise 5. *Change of variables.* Let t be as below.

$$\int \sqrt{x}\cdot e^{\sqrt{x}}\,dx \;\Rightarrow\; t = \sqrt{x} \qquad \int \frac{x^2}{(2x+4)^4}\,dx \;\Rightarrow\; t = 2x+4$$

$$\int \tan^2 x\,dx \;\Rightarrow\; t = \tan x \qquad \int \frac{e^x}{e^{2x}+4e^x+4}\,dx \;\Rightarrow\; t = e^x$$

Exercise 6. *Riemann integral.*

$\int_{-5}^{5} |x|\cdot e^x\,dx \;\Rightarrow$ we divide the interval based on the definition of the absolute value function and calculate both terms with integration by parts

$\int_{1}^{8} \sqrt[3]{x} - \frac{1}{2\sqrt[3]{x}}\,dx \;\Rightarrow$ we take $t = \sqrt[3]{x}$ (change of variables)