Mathematics 2
Exercises

1 Derivatives of functions of two variables

1.1. Compute both first order partial derivatives of the functions below.

(a) $f(x,y) = 4x^2 + 3xy + 5y^3$

(b) $f(x_1,x_2) = (3x_1 + 2x_2) \cdot \sin x_1$

(c) $f(x,y) = \frac{3x^2+7y^5}{x^3-y^2}$

(d) $f(x,y) = e^{x^2-8y}$

(e) $f(x,y) = x^2 \cdot \cos(3x+2y)$

(f) $f(x_1,x_2) = \ln \sqrt{x_1^2 + x_2^2}$

1.2. Determine the local extremum points of the following functions, and decide if they are local minimum or maximum points.

(a) $f(x,y) = 3xy - x^3 - y^3$

(b) $f(x,y) = x^2 + 2y^2 - x - 2y - 1$

(c) $f(x,y) = x^2y - 3xy + 2y^4$

(d) $f(x,y) = \frac{20}{x} + \frac{50}{y} + xy$

(e) $f(x,y) = 2x^2 + 2xy + y^2 - 7$

(f) $f(x,y) = \frac{y^3}{3} + x^2 - 2xy + 4$

(g) $f(x,y) = x^2y - 3xy + 3y^3$

1.3. Find the equation of the tangent plane to the surface given by

$$f(x,y) = 4x^2 + 3xy + 5y^3$$

at the point $(x_0,y_0) = (2,0)$.

2 Integration of functions of one and two variables

2.1. Find the following integrals.

(a) $\int t^2 - 6t + 5\,dt$

(b) $\int \frac{1}{x^2}\,dx$

(c) $\int \frac{x+\sqrt{x}+1}{\sqrt{x}}\,dx$

(d) $\int 2e^x + \frac{5}{x} + \frac{1}{\sin^2 x} + 3^x\,dx$

(e) $\int \frac{t-3}{t^2} + \sqrt[3]{t}\,dt$

(f) $\int e^{8x}\,dx$

(g) $\int \sin(3x-5)\,dx$

(h) $\int \frac{1}{4-7x}\,dx$

(i) $\int (2x-3)^{100}\,dx$

(j) $\int \frac{1}{\sqrt{8-x}}\,dx$

(k) $\int \cos^6 x \cdot \sin x\,dx$

(l) $\int \frac{x+1}{x^2+2x-1}\,dx$

(m) $\int \frac{3x}{(2+3x^2)^3}\,dx$

(n) $\int (x-3)\cos x\,dx$

(o) $\int (x^2-2x+10)e^{2x}\,dx$

(p) $\int (2x^3+6x^2-x+3)\ln x\,dx$

(q) $\int \ln x\,dx$

(r) $\int \frac{x^3}{(x+2)^4}\,dx$

(s) $\int \frac{1}{\sqrt{x+1}+\sqrt{x+1}^3}\,dx$

(t) $\int \frac{e^{3x}}{e^x+1}\,dx$

2.2. Find the value of the Riemann integrals below.

(a) $\int_0^{2\pi} \sin x \, dx$

(b) $\int_1^5 |x-2| \, dx$

(c) $\int_0^3 (x-1)e^x \, dx$

(d) $\int_0^1 \frac{1}{e^x+e^{-x}+2} \, dx$

2.3. Find the value of the following Riemann integrals.

(a) $\displaystyle\int_{[0,2]\times[0,1]} xy^2 \, d(x,y) = \int_{[0,1]} \int_{[0,2]} xy \, dx \, dy = \int_{[0,2]} \int_{[0,1]} xy \, dy \, dx$

(b) $\displaystyle\int_{[-1,1]} \int_{[0,2]} e^{x+y} \, dx \, dy = \int_{-1}^{1} \int_0^2 e^{x+y} \, dx \, dy$

(c) $\displaystyle\int_0^1 \int_0^{e^x} 5 - 4y + x \, dy \, dx$

(d) $\displaystyle\int_A (x^2+y^2) \, d(x,y)$, where $A = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le 1, 0 \le y \le x^2\}$

(e) $\displaystyle\int_{-1}^{2} \int_0^{2-y} 8 - \cos(x-y) \, dx \, dy$

(f) $\displaystyle\int_A x + y \, d(x,y)$, where A is the set of $\mathbb{R}^2$ bounded by the lines $y = 2 - 3x$ and the 2 axes

3 Vector spaces – $\mathbb{R}^n$

3.1. Solve the following vector equation. What is (x_1, x_2, x_3) ?

$$3(x_1, x_2, x_3) + 5(-2, 1, 0) = (-1, 2, 12)$$

3.2. Show that the vectors $(1,2)$ and $(-3,2)$ are linearly independent in $\mathbb{R}^2$.

3.3. Are the vectors below linearly dependent or independent in $\mathbb{R}^3$?

(a) $v_1 = (1,0,1)$, $v_2 = (1,1,1)$;
(b) $v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (1,1,1)$;
(c) $v_1 = (1,2,2)$, $v_2 = (-1,-1,2)$, $v_3 = (2,3,0)$;
(d) $v_1 = (1,1,2)$, $v_2 = (-2,-2,-4)$;
(e) $v_1 = (-1,2,1)$, $v_2 = (2,-3,1)$, $v_3 = (1,1,2)$;
(f) $v_1 = (-1,2,1)$, $v_2 = (2,-3,1)$, $v_3 = (1,1,2)$, $v_4 = (5,2,0)$.

3.4. What are the coordinates in $\mathbb{R}^2$ of the vector $(1,2)$ with respect to the bases below?

(a) $(1,0)$, $(0,1)$;
(b) $(-1,2)$, $(1,1)$;
(c) $(3,1)$, $(1,2)$;
(d) $(1,-1)$, $(1,5)$.

3.5. Is it possible to write the vector $(1,2) \in \mathbb{R}^2$ as $\lambda_1 v_1 + \lambda_2 v_2$ with some scalars λ_1, λ_2, where $v_1 = (3,1)$, $v_2 = (-3,-1)$?

3.6. What are the coordinates in $\mathbb{R}^3$ of the vector $(1,-1,2)$ with respect to the bases below?

(a) $(1,0,0)$, $(0,1,0)$, $(0,0,1)$;
(b) $(1,0,0)$, $(0,1,0)$, $(1,1,1)$;
(c) $(-1,2,1)$, $(2,-3,1)$, $(1,1,2)$.

3.7. Consider the vectors v and w below. Is it possible to write w as λv with some $\lambda \in \mathbb{R}$?

(a) $\mathbb{R}^2$: $v=(1,-3)$, $w=(-2,6)$.
(b) $\mathbb{R}^2$: $v=(3,5)$, $w=(-4,1)$.
(c) $\mathbb{R}^2$: $v=(2,7)$, $w=(0,0)$.
(d) $\mathbb{R}^2$: $v=(0,0)$, $w=(2,7)$.
(e) $\mathbb{R}^3$: $v=(3,12,-9)$, $w=(1,4,-3)$.
(f) $\mathbb{R}^3$: $v=(3,3,0)$, $w=(-6,-6,1)$.

4 Matrices, determinants

4.1. Consider the matrices A, B, C and D:

$$A=\begin{pmatrix} 4 & -1 & 0 \\ 1 & 5 & -3 \end{pmatrix} \quad B=\begin{pmatrix} 0 & 3 & -2 \\ 1 & 1 & 4 \end{pmatrix} \quad C=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & 10 \\ 4 & 0 & 4 \end{pmatrix} \quad D=\begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}$$

Do the following operations if possible.

$$A+B,\ 3A-2B,\ A^T+B^T,\ AD,\ CB,\ CD,\ AC+BC,\ (A+B)C$$

4.2. Let

$$v=\begin{pmatrix} 4 \\ -3 \end{pmatrix},\quad w=\begin{pmatrix} 0 \\ -1 \end{pmatrix},\quad A=\begin{pmatrix} -1 & 3 \\ 2 & -4 \end{pmatrix},$$
$$B=\begin{pmatrix} 4 & 2 \\ -2 & -3 \end{pmatrix},\quad C=\begin{pmatrix} 2 & -3 \\ -3 & -5 \end{pmatrix},\quad E=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Calculate the matrices below.

$$Av,\quad Aw\quad A(u+v),\quad w^TA^T,\quad A+B,\quad (-2C)^T,\quad 2A-3B,\quad (A+B)^T,\quad AB,\quad BA,$$
$$B^TA^T,\quad AE,\quad EA,\quad AC,\quad A(B+C),\quad C(A-2B),\quad A^TA,\quad AA^T.$$

4.3. Calculate the determinants below with the rule of Sarrus.

(a) $\begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix}$

(b) $\begin{pmatrix} 5 & -2 \\ 2 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 0 & 0 \\ -5 & 1 & 2 \\ 3 & 8 & -7 \end{pmatrix}$

(f) $\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

4.4. Calculate the determinants below with Laplace expansion or Gaussian elimination.

(a) $\begin{pmatrix} -3 & 4 \\ -2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & -1 & 2 \\ 0 & 0 & 5 \\ 0 & 1 & -5 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 1 & -1 \\ -6 & -6 & 5 \\ 4 & -4 & 3 \end{pmatrix}$ (e) $\begin{pmatrix} -3 & 0 & -1 \\ -1 & -6 & 5 \\ 2 & 0 & -3 \end{pmatrix}$ (f) $\begin{pmatrix} 2 & -1 & 1 \\ 4 & -2 & 5 \\ -2 & 1 & -5 \end{pmatrix}$

(g) $\begin{pmatrix} 5 & 1 & 2 & 7 \\ 3 & 0 & 0 & 2 \\ 1 & 3 & 4 & 5 \\ 2 & 0 & 0 & 3 \end{pmatrix}$ (h) $\begin{pmatrix} -2 & 2 & 0 & 1 \\ 4 & -2 & -1 & 1 \\ -4 & -2 & 4 & -8 \\ 8 & -6 & 1 & 0 \end{pmatrix}$ (i) $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix}$

4.5. Determine the inverse of the matrices below if it exists.

(a) $\begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 2 & -12 \\ -1 & 0 & 2 \\ 2 & 2 & -10 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & -1 & 2 \\ 1 & -3 & -4 \\ 2 & 2 & 5 \end{pmatrix}$ (e) $\begin{pmatrix} -2 & 1 & 4 \\ 1 & 0 & 3 \\ -1 & 2 & -3 \end{pmatrix}$ (f) $\begin{pmatrix} -1 & 2 & 1 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

(g) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (h) $\begin{pmatrix} -1 & 3 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & -3 \end{pmatrix}$ (i) $\begin{pmatrix} 1 & -3 & 2 \\ -6 & 4 & -1 \\ 1 & 3 & -6 \end{pmatrix}$

5 Systems of linear equations

5.1. Solve the following systems of linear equations if possible.

(a) $$\begin{aligned} x_1 - x_2 + x_3 &= 6 \\ 2x_1 + x_2 - x_3 &= 5 \\ -3x_1 + 2x_2 + 5x_3 &= 3 \end{aligned}$$

(b) $$\begin{aligned} 2x_1 + x_2 - x_3 &= 5 \\ x_1 - x_2 + x_3 &= 6 \\ x_1 + 2x_2 - 2x_3 &= -1 \end{aligned}$$

(c) $$\begin{aligned} x_1 + 4x_2 &= 5 \\ 2x_1 + 8x_2 &= 10 \end{aligned}$$

(d) $$\begin{aligned} x_1 - x_2 + 3x_3 + 5x_4 &= 2 \\ 2x_1 + 2x_2 \qquad\quad + x_4 &= 0 \\ -x_1 - 3x_2 + 3x_3 + 4x_4 &= 2 \end{aligned}$$

(e) $$\begin{aligned} -3x_1 + x_2 + 2x_3 &= -2 \\ 4x_1 - 6x_2 - x_3 &= 17 \\ x_1 - 5x_2 + x_3 &= 10 \end{aligned}$$

(f) $$\begin{aligned} x + y + z &= 3 \\ x + 2y + 2z &= 5 \\ 3x + 5y + 6z &= 14 \\ 2x + 4y + 5z &= 11 \\ x + 2y + 3z &= 6 \end{aligned}$$

5.2. Solve the following systems of linear equations if possible.

(a) $$\begin{aligned} -2x_1 - x_2 + 4x_3 &= 3 \\ 2x_1 + 3x_2 - x_3 &= 1 \\ -4x_1 - 10x_2 - 5x_3 &= -12 \end{aligned}$$

(b) $$\begin{aligned} -4x_1 - 4x_2 + 2x_3 &= -2 \\ -2x_1 - 7x_2 + 3x_3 &= 6 \\ 2x_1 + 12x_2 - 5x_3 &= -13 \end{aligned}$$

(c) $\begin{aligned} -2x_1 + 2x_2 + x_3 &= 0 \\ 6x_1 - 3x_2 - 4x_3 &= -8 \\ -4x_1 + x_2 + x_3 &= 4 \end{aligned}$

(d) $\begin{aligned} 3x_1 + 2x_2 - 4x_3 - x_4 &= 4 \\ -x_1 - 2x_2 + 3x_3 + 5x_4 &= -5 \\ 2x_1 + 4x_2 - 2x_3 - 5x_4 &= 8 \end{aligned}$

(e) $\begin{aligned} 2x_1 - 4x_2 + 2x_3 &= -2 \\ -4x_1 + 6x_2 - x_3 &= 5 \\ x_1 - 2x_3 &= 0 \end{aligned}$

(f) $\begin{aligned} -x_1 + 3x_2 - 2x_3 &= 3 \\ 2x_1 - 4x_2 + 9x_3 &= 3 \\ -x_1 + 7x_2 + 7x_3 &= 20 \\ x_1 - 5x_2 - 2x_3 &= -11 \end{aligned}$

5.3. What is the rank of the matrices below?

$$\begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & -4 \end{pmatrix}, \quad \begin{pmatrix} 3 & 1 & -1 \\ -6 & -2 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 0 \\ -2 & 2 & -6 \\ 0 & 4 & -4 \\ 3 & -3 & 9 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{pmatrix}, \quad \begin{pmatrix} 3 & 2 & 1 & -1 \\ -9 & -8 & -1 & 6 \\ -6 & 0 & -8 & -8 \\ 9 & 8 & 7 & 5 \end{pmatrix}, \quad \begin{pmatrix} -2 & 4 \\ 1 & 3 \\ 0 & 2 \\ -5 & 2 \end{pmatrix}$$

6 Euclidean vector spaces

6.1. Calculate the norms, the scalar product and the angle of the two vectors below (with respect to the canonical scalar product).

(a) $v = (1, 2, -7)$ and $w = (0, 4, 1)$;

(b) $v = (8, 1, -1)$ and $w = (0, 2, 1)$;

(c) $v = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $w = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

7 Combinatorics

7.1. There are 10 participants at a running competition. How many different orders can they finish in?

7.2. There are 10 participants at a running competition. How many possibilities are there for the podium (that is, for the first 3 places)?

7.3. You are eating at Emile's restaurant and the waiter informs you that you have (a) two choices for appetizers: soup or juice; (b) three for the main course: a meat, fish, or vegetable dish; and (c) two for dessert: ice cream or cake. How many possible choices do you have for your complete meal?

7.4. How many actual four digit numbers (they can not start with zero) can be formed from the digits 0, 1, 2, 3, 4, 5, 6?

7.5. Are there more

- 9-digit numbers with all different digits,
- or 10-digit numbers with all different digits?

7.6. We have 12 books on the shelf. How many ways can the books be arranged on the shelf if 3 particular books have to be next to each other

(a) if the order of the three books does not count?
(b) if the order of the three books does count?

7.7. In how many ways can one arrange 4 math books, 3 chemistry books, 2 physics books, and 1 biology book on a bookshelf so that all the math books are together, all the chemistry books are together, and all the physics books are together?

7.8. Find the number of possible arrangements of 8 castles on the chess board in a way that they do not hit each other? What is the result if we can distinguish between the castles?

7.9. In a box there are 5 red, 7 blue and 3 green balls. We take them out one by one, and write down the colors in order. How many possibilities are there for the sequence of colors?

7.10. There is a group of 10 going for a journey, and at the hotel they booked a room with 2 beds, a room with 3 and a room with 5 beds. How many possibilities are there for distributing the group among the rooms?

7.11. There are 15 (different) books and 12 children. In how many ways can we distribute the books among the children?

7.12. There are 15 apples and 12 children. In how many ways can we distribute the apples among the children?

7.13. Three postmen has to deliver six letters. Find the number of possible distributions of the letters.

7.14. In how many ways can 7 people be arranged around a round table?

7.15. In how many ways can we arrange 5 couples around a round table if the members of each couple want to sit next to each other?

7.16. In how many ways can 5 men and 5 women be arranged around a round table if neither two men, nor two women can sit next to each other?

7.17. In how many ways can one fill a toto coupon (14 matches, 3 possible results: 1, 2 or X)?

7.18. Find the number of possible choices of four cards of four different colours from a deck of ordinary cards (4 colours, 13 cards per colour). What is the result if we require that the four cards should be of different figures as well?

7.19. Find the number of possible paths from the origin to the point $(5, 3)$ if we can walk only on points with integer coordinates and we can step only upwards and right.

7.20. In Circus Maximus the tamer has to lead 5 lions and 4 tigers to the ring, but tigers can not follow each other because they fight. In how many ways can he do it if the animals can be distinguished?

7.21. Find the number of possible arrangements of 4 boys and 6 girls such that there are no boys standing next to each other.

7.22. A deck of ordinary cards is shuffled and 10 cards are dealt.

(a) In how many cases will we have aces among the 10 cards?
(b) Exactly one ace?
(c) At most one ace?
(d) Exactly two aces?
(e) At least two aces?

7.23. In how many ways can we choose four dancing pairs from 12 girls and 15 boys?

7.24. Find the number of 5-digit numbers having 3 odd and 2 even digits.

7.25. In the canteen we can by four types of snacks. In how many ways can we buy 12 of them?

7.26. In how many ways can we choose a group of 4 from 5 boys and 5 girls, such that there are at least two girls among them?

7.27. Prove that

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n.$$

8 Events in a probability space

8.1. A coin is tossed. If the result is a head, it is tossed once again, otherwise it is tossed twice again. Give the sample space of the experiment.

8.2. A dice is thrown three times. Let A_i denote the event that the result of the ith throw is 6, $i = 1, 2, 3$. What is the meaning of the following events:

$$A_1 + A_2, \quad A_1 \cdot A_2, \quad A_1 + (A_2 \cdot A_3), \quad A_1 \cdot \overline{A_2}, \quad A_1 \setminus A_2 \;?$$

9 Classical probability space

9.1. Two fair dice are thrown. Find the probability that the sum of the numbers obtained is 8. Illustrate the sample space and the set of favourable events.

9.2. Ten coins are tossed. Find the probability that all of them show head or all of them show tail.

9.3. From a deck of cards three cards are dealt. Find the probability that there isn't any spade among them.

9.4. In a dark room we have four pairs of the same shoes mixed. Find the probability that if four shoes are chosen we have at least one pair among them.

9.5. In an urn we have three red balls. Find the minimal number of white balls to be added to have the probability of choosing a white ball be greater than 0.9.

9.6. Find the probability that on the lottery 5 from 90 we hit at least three winning numbers.

9.7. In an urn we have 3 red, 3 white and 3 green balls. Find the probability of having all three colours among 6 randomly chosen balls.

9.8. What is the probability that two members in a group of four have their birthdays on the same day (365 days of a year considered)?

9.9. From 40 questions a student learned just 20. On the exam he has to choose randomly two questions, but then he is free to choose one of the two to work on it. Find the probability that he passes the exam.

10 Conditional probability, Bayes' theorem

10.1. Two dice are rolled. Find the probability that at least one of them shows six, given they show different values.

10.2. We know that at least one of the two kids in a family is a girl. Find the probability of having also a boy in the family.

10.3. From a box containing 5 red and 5 white balls 3 balls are chosen without replacement. Given the first two chosen balls are of the same colour find the probability that the third chosen ball is red.

10.4. There are six six-shooter revolvers laying on a table. Three of them are loaded with 1-1 bullets, two of them with 2-2 bullets and the sixth is with 3 bullets. We chose randomly a revolver and we pull the trigger. Find the probability that the chosen revolver shots.

10.5. Consider the revolvers of the previous exercise. Given a randomly chosen revolver shots, find the probability that no more bullets are left in the chamber.

10.6. In an office equipped with mechanized administration three machines classify the files. The first can process 10 files per day, the second 15, while the third 25. The average numbers of misclassified files are 0.3, 0.9 and 0.5 per day, respectively. We choose a file randomly from the daily production and we find that it has been misclassified. What is the probability that the file was processed by the first machine?

10.7. The overseas flights of the Hornet Airways are operated with aircrafts of types D, E and F, all types fly with the same probability. On type D there are six seats per row, on type E four, on type F three (each row has two window seats) and the passengers get their seats randomly. Given you have a window seat find the probability that you are flying with type F.

11 Discrete random variables

11.1. Two dice are rolled. What are the distributions of the minimum and of the maximum of the numbers obtained?

11.2. A coin is tossed. If the result is a head, it is tossed one more time, otherwise it is tossed two more times. What is the mean number of heads obtained?

11.3. A blind knife thrower hits the target with probability $1/4$ and he keeps trying till the first hit.

(a) What is the distribution of the number of required trials? (Geometric distribution.)

(b) Find the mean and the standard deviation of the required trials.

11.4. A dice is rolled one hundred times. Find the mean and the variance of the sum of the numbers obtained.

11.5. Anna and Kate are playing tennis. Anna wins a game with probability 0.4, they play 3 games, and the winner is the player with more games won.

(a) What is the probability that Anna wins the tennis party?

(b) What is the expected value of the number of games won by Anna?

11.6. Two dice are rolled till the first six appears on one of them. What is the expected number of rolls required, inclusive the last one? (Geometric distribution.)

11.7. The values of a random variable ξ are $-1, 0, 2, 3$, while the corresponding probabilities are $\frac{1}{12}$, $\frac{5}{12}$, $\frac{1}{4}$, $\frac{1}{4}$, respectively. Find the mean and the standard deviation of ξ^2.

11.8. Two dice are rolled. Find and plot the cumulative distribution function (cdf) of the sum of the numbers obtained.

11.9. In a certain zoo there are two sloths. Every day the first sleeps in half of the time while the second in one third of the time, independently of the first. Let ξ denote the number of sloths that are awaken during our visit in the zoo. Find the cdf of ξ.

12 Absolutely continuous random variables

12.1. Check whether the following functions are probability density functions or not.

(a)

$$f(x) = \begin{cases} \frac{\sin(x)}{2}, & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(b)

$$f(x) = \begin{cases} \frac{1}{x^2}, & \text{if } x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

12.2. The pdf of a random variable ξ equals

$$f(x) = \begin{cases} 0, & \text{if } x \leq 2, \\ \frac{A}{x^3}, & \text{if } x > 2. \end{cases}$$

(a) Find the value of A.

(b) Find the probability that $\xi \in [1, 4]$.

(c) Find the cdf of ξ.

(d) Determine the expected value of ξ.

(e) Show that the variance of ξ does not exist.