Discrete mathematics

Test 1 – sample

I. Theory

Exercise A Enumerate the Peano axioms.

Exercise B Define prime and composite numbers. When do we say that two integers are coprime numbers?

Exercise C What does Euler's theorem state? How do you compute $\varphi(m)$ for a natural number m, where φ is Euler's φ function?

Exercise D What is the formula for the nth roots of a complex number?

II. Practice

Exercise 1 Let us consider the sets $A = \mathbb{Z}$, $B = \{x \in \mathbb{Z} \mid x \text{ is even}\}$, $C = \{0, 1, 2, 3, 4\}$, $D = \{2, 3, 5, 7, 11, 13, 17\}$. What are the sets below?

$$A \cap B, \qquad C \setminus B, \qquad (A \setminus B) \cup D, \qquad B \triangle D$$

Exercise 2 Plot the function $f(x) = 2x^2 + 8x - 10$. Decide whether it is injective, surjective, bijective.

Exercise 3 Prove the following relation by mathematical induction.

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2 \qquad \forall n \in \mathbb{N}$$

Exercise 4 Prove the divisibility $6 | (10^7 - 88)$.

Exercise 5 Calculate the greatest common divisor of -845 and 680 with the Euclidean algorithm.

Exercise 6 Solve the linear congruence equation $9x \equiv 15 \pmod{12}$ if possible.

Exercise 7 Give the algebraic or trigonometric form of the complex numbers below.

$$i^7 - i^5 + 4i^4 + 20i^3 - 3i^2 + 5 = ? \qquad \left[3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^4 = ?$$

Exercise 8 Compute the fourth roots of the complex number $z = 81\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$.