How to calculate the determinant?

The rule of Sarrus: only for 2×2 and 3×3 determinants

Gaussian elimination (or row reduction): The modifications below doesn't change the determinant.

(1) If we multiply the determinant by a nonzero scalar, instead of multiplying all elements of a fixed row by the same scalar.

(2) If we add a scalar multiple of a a row to another row.

(3) If we interchange two rows, the determinant changes sign.

With the help of them we make our determinant to be upper triangular, then the determinant is the product of the elements in the main diagonal.

Laplace expansion: We choose an arbitrary row or column of the determinant. E.g., if we choose the i^{th} row, then

$$\det(A) = |A| = \sum_{j=1}^{n} a_{ij} C_{ij}, \quad \text{where}$$

- C_{ij} is the cofactor of A corresponding to the element a_{ij}, that is,

$$C_{ij} = (-1)^{i+j} A_{ij},$$

- A_{ij} is the $(n-1) \times (n-1)$ determinant obtained from A by deleting the i^{th} row and j^{th} column of A.

Solving a system of linear equations

Solution with Gaussian elimination: The set of solutions of a system of linear equations does not change, if we

(1) multiply an equation by a nonzero constant;

(2) add a scalar multiple of an equation to another equation;

(3) interchange two equations;

(4) discard an equation which is a scalar multiple of another equation.

We annihilate the numbers under the main diagonal with the modifications above. The resulting system is easier to solve.

- If during the process we obtain a row like $(0 \ldots 0 | \neq 0)$, then the system of linear equations is overdetermined. Thus, there are no solutions.
- Let n denote the number of the unknown parameters. If at the end of the process there are n number of rows, then the system is determined, thus, there is exactly 1 solution. If fewer number of rows remain, then undetermined, thus, there are infinitely many solutions.