

Systems of linear equations – Homework

Exercise Solve the following systems of linear equations.

$$(1) \quad \begin{aligned} x_1 + 2x_2 &= 1 \\ 2x_1 - 5x_2 &= -7 \end{aligned}$$

$$(2) \quad \begin{aligned} x_1 + 4x_2 &= 5 \\ 2x_1 + 8x_2 &= -1 \end{aligned}$$

$$(3) \quad \begin{aligned} 2x_1 + 9x_2 - x_3 &= -3 \\ x_1 + 4x_2 - 3x_3 &= 1 \\ -2x_1 - 10x_2 + 16x_3 &= 8 \end{aligned}$$

$$(4) \quad \begin{aligned} 3x_1 + 5x_2 - 12x_3 &= 10 \\ x_1 + 2x_2 - 5x_3 &= 2 \\ -x_1 + x_2 - 4x_3 &= -14 \end{aligned}$$

$$(5) \quad \begin{aligned} 3x + 2y - 12z &= 1 \\ -x + 2z &= 1 \\ 2x + 2y - 10z &= 2 \end{aligned}$$

$$(6) \quad \begin{aligned} -3x_1 + x_2 + 2x_3 &= -2 \\ 4x_1 - 6x_2 - x_3 &= 17 \\ 3x_1 + x_2 - 6x_3 &= -14 \end{aligned}$$

$$(7) \quad \begin{aligned} 2x + 3y + 5z &= 13 \\ x + 2y + 4z &= 10 \\ 2x + y + 3z &= 11 \\ x + y + z &= 3 \\ 3x + 2y + z &= 5 \end{aligned}$$

$$(8) \quad \begin{aligned} -x_1 + 3x_2 + 2x_3 - 2x_4 &= -1 \\ 3x_1 - 7x_2 + x_3 + 4x_4 &= 7 \\ -3x_1 + 5x_2 - 8x_3 - 2x_4 &= -11 \end{aligned}$$

Solutions:

$$(1) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2) \text{ inconsistent/overdetermined} \quad (3) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 21 \\ -5 \\ 0 \end{pmatrix}$$

$$(4) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \quad (5) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$(6) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} \quad (7) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$$

$$(8) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -\frac{17}{2} \\ -\frac{7}{2} \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ 0 \\ 0 \end{pmatrix} + s_1 \begin{pmatrix} -17 \\ -7 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$